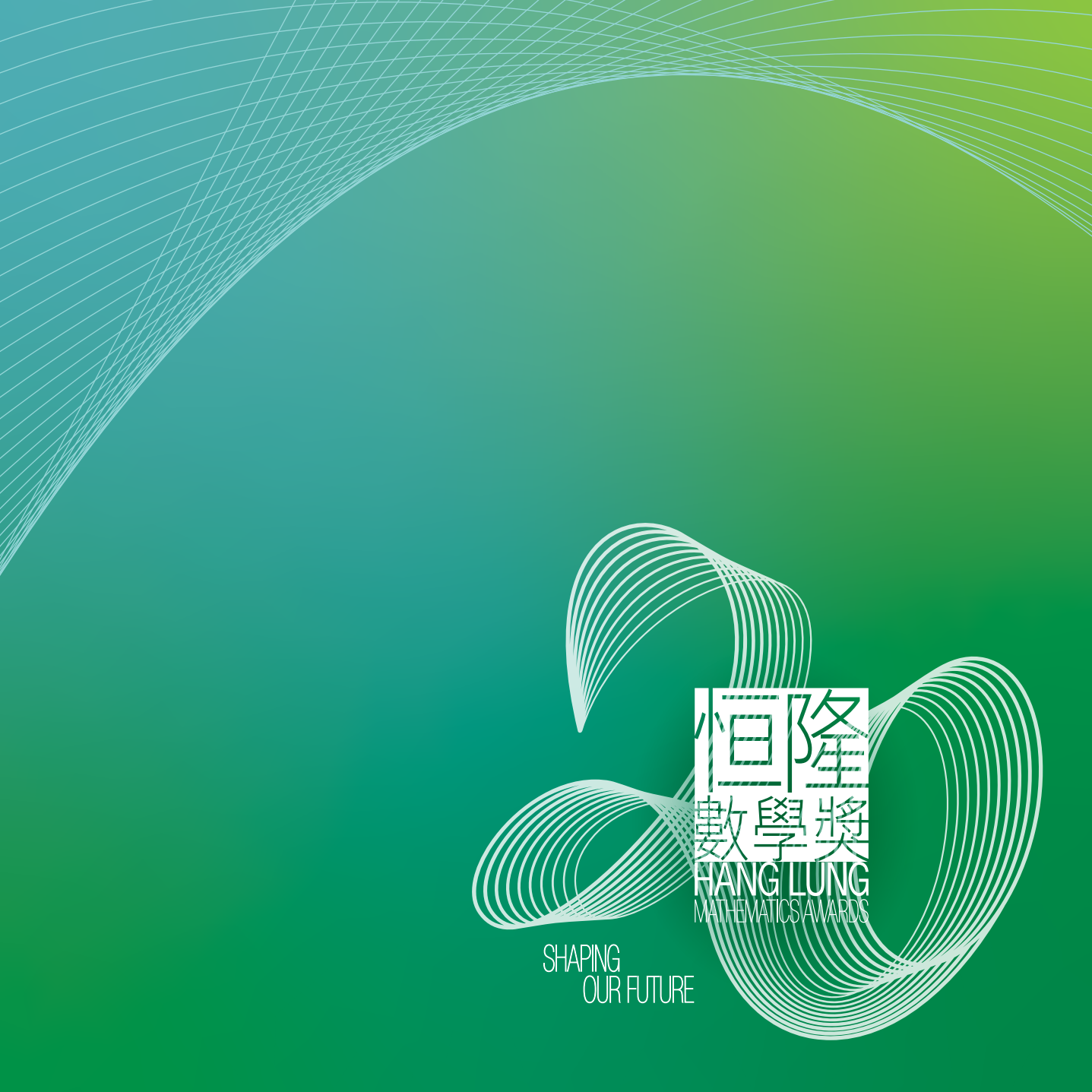

恒隆
數學獎
HANG LUNG
MATHEMATICS AWARDS
SHAPING
OUR FUTURE

2023 Hang Lung Mathematics Awards Announcement and Awards Presentation

Shaping our Future through Mathematics
A Celebration of 20 Years of Excellence

December 19, 2023
Conrad Hong Kong
Pacific Place, 88 Queensway, Hong Kong

Program

Welcome Remarks

Mr. Ronnie C. Chan	Chair Hang Lung Properties Limited
Professor Yike Guo	Provost The Hong Kong University of Science and Technology

Opening Remarks

The Hon. John KC LEE GBM, SBS, PDSM, PMSM	The Chief Executive Hong Kong Special Administrative Region

Dinner

Fireside Chat: Unleashing Possibilities in Neuroscience and Neural Engineering with Mathematics

Dr. Owen H. Ko	Assistant Dean (Research), Faculty of Medicine The Chinese University of Hong Kong
Ms. Ewina Pun	Doctoral candidate Brown University
Mr. Adriel Chan (moderator)	Vice Chair Hang Lung Properties Limited

Vote of Thanks

Mr. Weber Lo	Chief Executive Officer Hang Lung Properties Limited

Special Remarks

Professor Richard Schoen	Chair, Scientific Committee 2023 Hang Lung Mathematics Awards

2023 Hang Lung Mathematics Awards Announcement and Awards Presentation

Group Photos

Welcome Letter

Dear Friends,

A warm welcome to all of you as we gather here to announce the winners of the 2023 Hang Lung Mathematics Awards (HLMA) and celebrate 20 years of this esteemed competition. We are particularly pleased to have The Honorable John KC LEE, The Chief Executive of the Hong Kong Special Administrative Region, as our Guest of Honor and mark this significant milestone in our journey.

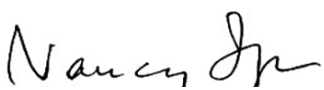
For two decades, HLMA has served as a testament to our commitment to the development of young minds in Hong Kong and support the city's vision as a global hub for innovation and technology. Since its inception in 2004, HLMA has ignited the intellectual curiosity of 2,600 secondary school students, resulting in over 460 research papers that showcase exceptional abilities in analytical thinking and problem solving. By investing in the cultivation of young talent, we are investing in the future of the city – one led by visionaries and innovators whose ideas will help strengthen Hong Kong's role on the global stage.

The challenges we face today are multidimensional and complex, ranging from rapid technological disruptions to widespread impact of climate change and intricate global dynamics. Addressing complexities of this magnitude requires sophisticated and holistic approaches. Mathematical training nurtures analytical thinking and problem-solving skills essential for comprehending complex systems through modeling uncertainties and simulating outcomes. By honing these skills at a young age, we lay the foundation for new generations to address pressing concerns and unlock new possibilities. These are invaluable, especially in times of disruptions and global challenges.

We are privileged to have Professor Richard Schoen, 2017 Wolf Prize Laureate in Mathematics, to serve as Chair of our Scientific Committee, and Professor George F. Smoot, 2006 Nobel Laureate in Physics, to chair our Steering Committee. These two principal committees, comprising distinguished mathematicians and educators from around the world, play a vital role in the integrity and stewardship of the awards.

As we celebrate this milestone, we extend our gratitude to all who have supported HLMA over the past 20 years. Together, through longstanding community programs like HLMA, we spark curious minds and provide opportunities for our youth to flourish. We look forward to continuing this important work that empowers future generations for many years to come.

Mr. Ronnie C. Chan
Chair
Hang Lung Properties Limited



Professor Nancy Ip
President and The Morningside Professor of Life Science
The Hong Kong University of Science and Technology

Welcome Letter

Dear Students, Teachers, and Honored Guests,

Welcome to the 2023 Hang Lung Mathematics Awards (HLMA) Announcement and Awards Presentation. Tonight, we celebrate not only the achievements of our students but also 20 years of our remarkable HLMA journey.

As we reflect on the past two decades, we are heartened to see the impact HLMA has made on young mathematics talents. HLMA fosters a culture of excellence and intellectual curiosity. It provides a platform for students to gain early exposure to academic research through explorations of mathematical concepts. We extend our heartfelt appreciation to the Department of Mathematics at The Hong Kong University of Science and Technology, which has provided invaluable guidance and support to students and teachers through a series of events and workshops delivered both online and in-person. These workshops have been instrumental in enriching the knowledge of the students and empowering them to excel.

As in our tradition of featuring HLMA alumni at the Awards Presentation, we are delighted to have Dr. Owen H. Ko, 2004 HLMA winner, and Ms. Ewina Pun, 2012 HLMA winner, join us tonight. Owen is Assistant Dean (Research) of the Faculty of Medicine at CUHK. Ewina is currently a doctoral candidate at Brown University focusing on brain-machine interface for people with tetraplegia. We look forward to their sharing of experiences and journey.

We would like to express our gratitude to the members of our Scientific Committee, Steering Committee, Executive Committee, and Screening Panel, who have contributed their time and expertise to support HLMA's vision. We are also grateful to the students, teachers, schools, and advocates whose participation and support over the years are strong endorsements of our work. To all students who accepted the challenge of pursuing serious mathematics research, we commend your courage in following your intellectual curiosity and persevering through difficulties. We hope you will continue to follow your passion and explore the vast world of mathematics.

This dedication and passion lay the foundation for promising opportunities ahead. Many former participants have made notable progress in academia, public sector, and various professions. We have faith that through HLMA's encouragement of curiosity, excellence, and perseverance, they are well equipped for future breakthroughs. Furthermore, many HLMA participants have later become educators, fostering a passion for research and exploration in their students, thereby generating a ripple effect that aligns with Hong Kong's vision of nurturing STEM talent and enhancing global competitiveness.

We take immense pride in the enduring impact of HLMA as a catalyst for excellence and a driving force in advancing mathematics. Together, we shape a future where young minds continue to push the boundaries of knowledge and make meaningful contributions to society.

In appreciation,

Mr. Weber Lo
Chief Executive Officer
Hang Lung Properties Limited

Professor Richard Schoen
Chair, Scientific Committee
2023 Hang Lung Mathematics Awards

Professor George F. Smoot
Chair, Steering Committee
2023 Hang Lung Mathematics Awards

Professor Yang Wang
Vice-President for Institutional Advancement
The Hong Kong University of Science and Technology

Hang Lung Mathematics Awards

Hang Lung Mathematics Awards is a highly regarded biennial mathematics research competition for secondary school students in Hong Kong. Founded in 2004 by Hang Lung Properties and The Chinese University of Hong Kong, Hang Lung Mathematics Awards has blazed a trail in secondary school mathematics education, encouraging students to realize their creative potential in mathematics and sciences, and stimulating their passion for intellectual discovery.

In 2021, Hang Lung Properties teamed up with one of the world's top young universities, The Hong Kong University of Science and Technology, as our partner in the next stage of Hang Lung Mathematics Awards' development. Under this partnership, Hang Lung Properties donates HK$2.5 million to each competition, of which HK$1 million is set aside as monetary prizes, and the remainder goes towards academic consultancy, assessment process, administration of the competition, as well as education promotion activities. In addition, The Hong Kong University of Science and Technology offers tuition scholarships for teachers nominated by the winning schools to enroll in the Master of Science Program in Mathematics for Educators.

Hang Lung Mathematics Awards has two principal committees, the Scientific Committee, the academic and adjudicating body of the competition, and the Steering Committee, an advisory body that provides support, guidance, and oversight of the Awards. Professor Richard Schoen, 2017 Wolf Prize Laureate in Mathematics, is the Chair of the 2023 Hang Lung Mathematics Awards Scientific Committee, which boasts 12 renowned scholars from eminent universities around the globe. Professor George F. Smoot, 2006 Nobel Laureate in Physics, chairs the 2023 Hang Lung Mathematics Awards Steering Committee, which comprises mathematicians, scientists, and leaders from different sectors of society.

Schools are invited to form teams of up to five students. Under the supervision of a teacher, each team decides on a mathematics topic, designs and carries out a mathematics research project, then submits a research report that summarizes the methodology, research, and results. The Scientific Committee will evaluate the research reports in a rigorous, multi-step review process that is comparable to publishing an article in a scientific journal and determine the teams that will be invited to participate in an oral defense. The oral defense is modeled after the doctoral dissertation defense, and comprises a brief public presentation of the research project, followed by a closed-door inquiry by the Scientific Committee. At the conclusion of the oral defense, the Scientific Committee will decide the winners of the Hang Lung Mathematics Awards, and the winners will be announced at the awards presentation ceremony.

Hang Lung Mathematics Awards

Over 70 teams from nearly 50 secondary schools in Hong Kong participated in the 2023 Hang Lung Mathematics Awards. At each Hang Lung Mathematics Awards, up to eight awards will be presented to recognize those mathematics research projects that meet the highest academic standards in terms of methodology, research, and scholarship. The eight awards are: a Gold Award, a Silver Award, a Bronze Award, and up to five Honorable Mentions.

Each award has four components: a Student Education Award, to be shared equally among team members and applied towards their university studies; a Teacher Leadership Award, for the supervising teacher; a School Development Award, for the winning school to promote mathematics education, and an Educator Scholarship, for a teacher nominated by the winning school. Besides monetary prizes, each winning student and teacher will receive a crystal trophy and a certificate, and their winning school will receive a crystal trophy.

Etched inside the crystal trophy is a visualized representation of an Einstein-Rosen bridge. An Einstein-Rosen bridge, sometimes called a "wormhole", is a transcendental bijection of the spacetime continuum, an asymptotic projection of the Calabi-Yau manifold manifesting itself in Anti-de Sitter space, first described by Ludwig Flamm in 1916 and rediscovered by Albert Einstein and Nathan Rosen in 1935. It represents the geometry of a "throat" that joins disparate regions of spacetime, but it is non-traversable, i.e., no physical timelike trajectory crosses the bridge. It plays a prominent role in mathematics and physics, where it has implications for geometry, gravity, information, and quantum physics.

Hang Lung Mathematics Awards					
in HK$	Student Education Award	Teacher Leadership Award	School Development Award	Educator Scholarship	Amount of each Award
1 x **Gold Award**	$250,000	$50,000	$100,000	Full program fee	$400,000 + Educator Scholarship
1 x **Silver Award**	$120,000	$20,000	$60,000	Full program fee	$200,000 + Educator Scholarship
1 x **Bronze Award**	$60,000	$10,000	$30,000	Full program fee	$100,000 + Educator Scholarship
5 x **Honorable Mentions**	$32,000	$8,000	$20,000	Half program fee	$60,000 + Educator Scholarship
Total	**HK$1,000,000 + Educator Scholarships**				

To learn more about Hang Lung Mathematics Awards, please visit our website and follow us on Facebook and Instagram.

2023 Hang Lung Mathematics Awards
Committees and Panel

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2017 Wolf Prize Laureate in Mathematics
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2023 Hang Lung Mathematics Awards
Committees and Panel

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2006 Nobel Laureate in Physics
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Mr. Alfred Cheung
NeoUnion ESC Organization

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Fung Kai Public School

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The Chinese University of Hong Kong

Professor Michael Kwok Po Ng
Hong Kong Baptist University

Professor Yang Wang
The Hong Kong University of
Science and Technology

Ms. Susan Wong
Hang Lung Properties Limited

Professor Tong Yang
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Dr. Chee Tim Yip
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2023 Hang Lung Mathematics Awards
Committees and Panel

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Professor Ivan Chi Ho Ip
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Professor Maosheng Xiong
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Dr. Albert Yin-Bon Ku
The Hong Kong University of
Science and Technology

Secretariat

Ms. Niki Hiu Yan Chan
The Hong Kong University of
Science and Technology

2023 Hang Lung Mathematics Awards
Finalists (listed by school name in alphabetical order)

1. School : Baptist Lui Ming Choi Secondary School
   Teacher : Ms. Wing Yan Lee
   Team Member : Hiu Long Chan
   Topic : Equilateral Polygons over Finite Fields

2. School : Diocesan Boys' School
   Teacher : Mr. Long Tin Chan
   Team Members : Pak Hei Chan, Yung Ho Kwan, Chi Ho Wang
   Topic : Investigating the Edge 1-2 Conjecture: from Complete Graphs to Trees and Unicyclic Graphs

3. School : Diocesan Boys' School
   Teacher : Mr. Long Tin Chan
   Team Members : Oscar Tin Yau Chan, Ernest Yik To Hsia, Ryan Cheuk Yi Lo, Tsz Yeung Ng
   Topic : On the Upper-bound of Anchored Packing Problem

4. School : Diocesan Boys' School
   Teacher : Mr. Long Tin Chan
   Team Members : Mason Chi Wah Chan, Tsz Sing Chong, Lucas Chavez Mocan, Himansh Vijay Pahilwani, Tsz Ho Wong
   Topic : Structure of Critical Groups of Circulant Graphs

5. School : G.T. (Ellen Yeung) College
   Teacher : Mr. Alexander Kin Chit O
   Team Member : Chun Lam Wong
   Topic : Quantum Computing: Adapting Shor's Algorithm to the Problem of CLT Group Size Finding

6. School : Harrow International School Hong Kong
   Teacher : Mr. Thomas Johnson
   Team Members : Kyan Ka Hin Cheung, Ethan Jon Yi Soh
   Topic : On the Properties of the Semigroup Generated by the RL Fractional Integral

7. School : HKUGA College
   Teacher : Mr. Yuk Lun Fong
   Team Members : Ho Lam Lau, Ryan Sze Chit Lo, Jane Chi Kuan Ng, Kin Long Wong
   Topic : Algorithmic Classification on the Expansion of Fractions in Negative Rational Base

8. School : HKUGA College
   Teacher : Mr. Yuk Lun Fong
   Team Member : Hei Tung Tam
   Topic : On the Parametrization of Egyptian Fractions

9. School : Kiangsu-Chekiang College (Kwai Chung)
   Teacher : Dr. Kit Wing Yu
   Team Member : Cho Wai Wong
   Topic : On Solutions of the Exponential Diophantine Equation $p^x - q^y = z^2$

10. School : P.L.K. Centenary Li Shiu Chung Memorial College
    Teacher : Mr. Yuet Hui
    Team Members : Wan Lau, Haopei Lin
    Topic : Generalization of Stern's Diatomic Sequence

11. School : Pui Ching Middle School
    Teacher : Mr. Yan Lam Fan
    Team Members : Wang Chit Kwong, Ngai Wang Lam, Ho Tung Wong
    Topic : Mean Shadow of Rotating Objects

12. School : S.K.H. Tsang Shiu Tim Secondary School
    Teacher : Mr. Tin Yau Cheung
    Team Members : Chun Hei Mok, Hei Wong
    Topic : Least Optimal Square Packing in a Square

13. School : St. Paul's Co-educational College
    Teacher : Mr. Pak Leong Cheung
    Team Members : Kwan Yu Chan, Hok Lai Ng
    Topic : Generalising Orthocentres of Triangles to Simplices as the Isogonal Conjugates of the Circumcentres

14. School : St. Paul's College
    Teacher : Mr. Tin Wai Lau
    Team Members : Ryan Chun Wing Ku, Lok Yung
    Topic : Covering 45 Points Configuration with Disjoint Unit Disks

15. School : Wah Yan College, Kowloon
    Teacher : Mr. Wai Man Chu
    Team Members : Hui Yat Cheung, Tsz Yan So, Yi Lam Tam
    Topic : On the Littlewood Problem and Sum of Two Squares in the Ring Z_q

1. Baptist Lui Ming Choi Secondary School

Equilateral Polygons over Finite Fields

The study of affine geometry over finite fields (as opposed to the rational, real or complex number fields) has been an interest among geometers. Most notably, the number of d-dimensional affine subspaces in a n-dimensional space over a finite field is a classic result dating back to the early 20th century, which involves the Gaussian binomial coefficient. Recently, N.J. Wildberger, in his book *Divine Proportions*, mentions the existence of a regular pentagon over a finite field of order 19. This has prompted a question: how many pentagons can be constructed over a finite field? More generally, how many equilateral polygons can be found, and how many are there up to symmetry? This paper investigates this problem, and gives both analytic and computational results on the number of equilateral polygons.

2. Diocesan Boys' School

Investigating the Edge 1-2 Conjecture: from Complete Graphs to Trees and Unicyclic Graphs

The 1-2-3 Conjecture, posed by Karoński, Luczak, and Thomason, questions whether for every connected graph G different from K_2, it is possible to assign a weight equal to 1, 2 or 3 to each edge such that for every two adjacent vertices in G, their sums of incident edge weights are distinct. Similar to the above problem, the Edge 1-2 Conjecture is to classify all graphs such that it is possible to assign a weight equal to 1 or 2 to each edge such that the above condition is satisfied. While the Edge 1-2 Conjecture is not true for all graphs G different from K_2, for instance complete graphs, we will prove the Edge 1-2 Conjecture holds for all connected graphs that can be obtained by removing one edge from a complete graph, and classify all unicyclic graphs that can fulfil the Edge 1-2 Conjecture. Finally, we present a more efficient way of solving the 1-2-3 Conjecture for the specific case of triangle-union graphs, compared to the algorithm presented for the general 1-2-3 Conjecture.

3. Diocesan Boys' School

On the Upper-bound of Anchored Packing Problem

The Lower-Left Anchored Rectangle Packing (LLARP) problem is well-studied in optimisation. Given a finite set of points P in the unit square $[0, 1]^2$, where $(0, 0) \in P$, we seek a set of rectangles S that has maximal area coverage, each of which has a lower-left corner anchored at a point $p_i \in P$. S satisfies three constraints: the rectangles are pairwise non-overlapping, completely lie within $[0, 1]^2$ and the sides are parallel to the coordinate axes.

This paper looks into a variant of LLARP, concerning convex N-gons, anchored at their lower-left vertex and with the two edges adjacent parallel to the coordinate axes such that a right angle is formed. We focus on finding the upper bound for some set of points in maximal coverage anchored packing.

4. Diocesan Boys' School

Structure of Critical Groups of Circulant Graphs

The critical group of a graph is defined as the torsion subgroup of the cokernel of the Laplacian matrix of the graph. In this paper, we investigated the critical groups of two classes of unitary circulant graphs, which are Cayley graphs on the group of integers modulo n, with connecting set being the set of units modulo n. The explicit group structure of such graphs when n is product of two distinct primes and when n is a prime power, are computed using Ramanujan Sums. Furthermore, we investigated the critical groups of circulant graphs with fixed connecting sets, and expressed one of the components of the group as the greatest common divisor of real and imaginary parts of Chebyshev Polynomials.

5. G.T. (Ellen Yeung) College

Quantum Computing: Adapting Shor's Algorithm to the Problem of CLT Group Size Finding

Shor's algorithm, first proposed by Peter Shor in 1994, is a well-known quantum algorithm for the discrete logarithm problem and for factoring large integers. Via running on a quantum computer, it is able to have a much better asymptotic runtime than state-of-the-art algorithms on classical computers. This report builds upon the Shor's algorithm to construct a quantum algorithm for finding the order of CLT groups (groups satisfying the converse of Lagrange's theorem) with significant advantage over classical algorithms.

6. Harrow International School Hong Kong

On the Properties of the Semigroup Generated by the RL Fractional Integral

For an operator A, it is sometimes possible to define e^{At} as an operator in and of itself provided it meets certain regularity conditions. Like $e^{\lambda x}$ for ODEs, this operator is useful for solving PDEs involving the operator A. In this paper, we discuss the semigroups generated by the fractional integral, an operator appearing in PDEs in increasingly many fields, over Bochner-Lebesgue spaces.

7. HKUGA College

Algorithmic Classification on the Expansion of Fractions in Negative Rational Base

This paper rigorously explores the expansion of fractions in the unorthodox number system with a rational negative base $-N_b/D_b$, building on the work of Lucia Rossi and Jörg M. Thuswaldner on multiple number representations in such a base. Our objective is to establish a finite number of recurring expansions, using our novel theories and algorithms. We introduce definitions and conditions for four types of expansions, and present two distinct proofs for the Complete Residue System Theorem, our first main theorem. Our Second Main Theorem outlines the bounds of terminating and recurring expansions in any number system, providing a method to compute all expansions for any fraction m/n. These findings provide a thorough examination of fraction representations in the negative rational base system, enhancing understanding of its intricate characteristics.

8. HKUGA College

On the Parametrization of Egyptian Fractions

This study explores Egyptian fractions, focusing on parametrization to construct a unified approach to open problems in this field. The paper introduces a symmetric parametrization for Egyptian fraction equations, demonstrating its effectiveness through three applications. It also investigates conjectures related to the shortest length of Egyptian expansion and the Generalized Erdős-Straus conjecture, and explores connections with semiperfect numbers. The research leverages geometry to transform Egyptian equations into a parametrized system, offering a novel perspective on tackling open problems with and within the field of Egyptian fractions.

9. Kiangsu-Chekiang College (Kwai Chung)

On Solutions of the Exponential Diophantine Equation $p^x - q^y = z^2$

In this paper, solutions of the exponential Diophantine equation $p^x - q^y = z^2$ are investigated. In fact, using modular arithmetic with new tricks, we are able to prove Theorem 1.1 that it has at most one solution $x, y, z \in \mathbb{N}$, where $p \equiv 3 \pmod 4$ and $q \equiv 1 \pmod 4$. Meanwhile, our Theorem 1.2 concerns the special case when $p \equiv 1 \pmod 4$ and $q = 5$. We prove in part (a) that it has at most one solution provided that p satisfies $p \equiv 3, 7 \pmod{10}$. Besides, part (b) guarantees that the solution of the particular equation $13^x - 5^y = z^2$ is unique and this fills the loophole of Burshtein's proof. Alternatively, another proof of this result is given and the main tools we employ are results from linear forms in logarithms of algebraic numbers developed by Baker and Davenport. Interestingly, with applications of primitive Pythagorean triples, we reveal in part (c) a new connection between solutions with even y and classical half-generalized Fermat numbers p.

10. P.L.K. Centenary Li Shiu Chung Memorial College

Generalization of Stern's Diatomic Sequence

Stern's diatomic sequence is defined as $a_1 = 1$, $a_{2k} = a_k$ and $a_{2k+1} = a_k + a_{k+1}$. It has many useful properties such as $\{ a_n / a_{n+1} \}_{n \geq 1}$ that runs through all positive rational numbers exactly one time. In this research, we generalize the coefficient of a_k and a_{k+1}, which can be any real number p, r, and s, and find 2 closed forms and some summation formulas of it. We also investigate the properties in number theory when p, r, and s are positive integers.

11. Pui Ching Middle School

Mean Shadow of Rotating Objects

In this paper, we conduct an analysis of the problem concerning the mean shadow cast by rotating objects. The original problem was introduced by Cauchy in 1832. He proposed solutions for the 2-D and 3-D scenarios in 1842 and 1850, respectively. In the original problem, the shadow was formed by orthogonal projection. In 2022, the problem was revisited under the 3-D scenario of a light source with finite distance above the rotation center. Instead of 3-D scenario, we focus on the 2-D case and generalize the problem by placing the light source arbitrarily. We derive explicit formulae of the mean shadow. With these formulae, we provide a numerical method to compute the mean shadow, which surpasses the conventional simulation.

12. S.K.H. Tsang Shiu Tim Secondary School

Least Optimal Square Packing in a Square

A lot of effort has been devoted into solving the famous Square Packing Problem, which investigates the minimum side length of a square container that can pack n unit squares. This involves the search for the most optimal packing for squares. The aim of this research is to investigate an opposite idea to the original problem. We delve into the least optimal packing of squares, i.e., finding the minimum side length of a square container that can contain all configurations of n unit squares.

By considering the idea of a rotating container, we have successfully found the solution to the case of two squares. At the end, by studying the classification of intersections between the configuration and the container, as well as harnessing analytical methods, we have found the exact solution to the general case of n squares.

13. St. Paul's Co-educational College

Generalising Orthocentres of Triangles to Simplices as the Isogonal Conjugates of the Circumcentres

In this paper, we have generalised the orthocentre of a triangle as the isogonal conjugate of the circumcentre of a simplex. Along this generalisation, we have also carried two intriguing properties of the orthocentre of a triangle over to higher dimensions, which says that the isogonal conjugate of the circumcentre of a simplex is either the incentre or an excentre of its pedal simplex, and is also the radical centre of the facetal circumhyperspheres of the simplex.

To this end, we have extended the scope of isogonal conjugation with respect to simplices to non-interior points through developing new algebraic and geometric characterisations for it. We have also obtained a higher-dimensional analogue of a curious property of isogonal conjugates with respect to triangles, which says that when both a point and its isogonal conjugate with respect to a simplex are projected onto the facets, the projections formed are co-hyperspherical.

14. St. Paul's College

Covering 45 Points Configuration with Disjoint Unit Disks

We study the conjecture of the non-existence of disjoint unit disks covering the 45 points configuration, proposed by Aloupis, Hearn, Iwasawa, and Uehara. We proved that if such covering exists, then the number of disks N used must be $3 \leq N \leq 5$. We also showed that if $N = 3$, then the number of points covered in the outermost circle must be (8, 7, 6) up to permutation.

15. Wah Yan College, Kowloon

On the Littlewood Problem and Sum of Two Squares in the Ring Z_q

Sum of two squares is a well-developed topic in number theory and J.E. Littlewood asked whether, for given unequal positive numbers h and k, there exist infinitely many n such that n, $n + h$ and $n + k$ are sums of two squares. Mong, Lai, and Mak showed a method to find such triples but n must be a perfect square and one of h, k, and $|h - k|$ must also be a perfect square.

In this paper, we developed an explicit formula for generating infinitely many n which need not be a perfect square such that n, $n + 1$, and $n + 3$ are sums of two squares. We also developed a formula to solve Littlewood Problem where there is no restriction on h or k and this result is found to be related to sum of two squares among Eisenstein integers.

Lastly, we looked at the cyclic ring Z_q: Harrington, Jones and Lamarche found the condition when not all elements in Z_q are sums of two nonzero squares. We found the number of elements which are sums of two nonzero squares in all such cases.

Fireside Chat: Unleashing Possibilities in Neuroscience and Neural Engineering with Mathematics

The latest developments in neuroscience and neural engineering hold a lot of promise. However, integrating these fields can be difficult because they are very complex subjects with different languages, methodologies, and objectives. Luckily, there is a tool that can help overcome these challenges — mathematics. In this fireside chat, two HLMA earlier winners, Dr. Owen H. Ko and Ms. Ewina Pun, will join Mr. Adriel Chan, Vice Chair of Hang Lung Properties, on stage to discuss some of the challenges of collaborating across disciplines, and explore the remarkable potential of mathematics in advancing our understanding of the brain and unlocking groundbreaking innovations in neuroscience and neural engineering. Join us for insightful exchanges of ideas that shape the future of these fields.

Dr. Owen H. Ko
Assistant Dean (Research)
Faculty of Medicine
The Chinese University of Hong Kong

Owen Ko is a clinician-scientist at The Chinese University of Hong Kong (CUHK). He holds medical degrees from CUHK, and a PhD in neuroscience from University College London. At UCL, he revealed several fundamental rules governing how neurons with different functional roles in the brain are connected.

He now leads a CUHK team specializing in therapeutics development for neurodegeneration based on counteracting brain aging. His team has been recognized by several awards, including a Croucher Innovation Award from the Croucher Foundation, and an Asian Young Scientist Fellowship from the Future Science Awards Foundation.

He attributes a significant part of his career achievements thus far to what he learnt from his early interactions with friends who are exceptionally talented in mathematics. Dr. Ko is a 2004 HLMA winner.

Ms. Ewina Pun
Doctoral candidate
Brown University

Ewina Pun is currently a doctoral candidate at Brown University. Her current area of research lies in the emerging field of neural engineering. Specifically, she focuses on developing stable closed-loop brain-computer interfaces for applications in clinical neuroscience.

Brain-computer interface is a promising technology that provides intuitive control for people with tetraplegia, restoring movement and communication by bypassing the damaged motor pathway.

Prior to Brown, she obtained a dual degree at the University of Southern California, majoring in electrical engineering and biomedical engineering. She is a 2012 HLMA winner.

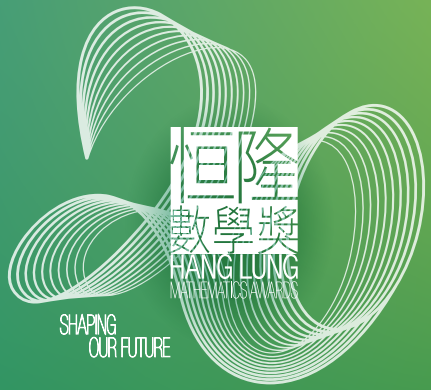
Mr. Adriel Chan (Moderator)
Vice Chair
Hang Lung Properties

Adriel Chan is the Vice Chair of Hang Lung Group and Hang Lung Properties. He joined in 2010, was appointed to the board in 2016, and appointed Vice Chair in 2020. Mr. Chan also chairs the sustainability committee.

Mr. Chan is a Vice President and Executive Committee member of the Real Estate Developers Association of Hong Kong and is Chairman of the Committee of Overseers of Morningside College at The Chinese University of Hong Kong. He sits on the Hong Kong University of Science and Technology (HKUST) Business School Advisory Council and is a Court Member of the University of Hong Kong. Additionally, Mr. Chan is a Council Member of the Academy of Chinese Studies.

Prior to joining the Group, Mr. Chan worked in finance, audit, and risk management. He holds an EMBA jointly awarded by the Kellogg School of Management at Northwestern University and HKUST, and a BA in international relations from the University of Southern California.

Shaping our Future through Mathematics
A Celebration of 20 Years of Excellence



To celebrate two decades of excellence, we proudly unveil a special logo inspired by geometric patterns in mathematics. The ten Bézier curves represent ten editions of the Awards. This logo symbolizes Hang Lung Mathematics Awards' commitment to excellence and the spirit of exploration that the competition instills in its participants, inspiring them to achieve their full potential through mathematics. Accompanying this logo is a slogan, "Shaping our Future," which encompasses our dedication to empowering young minds in mathematics and shaping a better future for all.

As Hang Lung Mathematics Awards enters its 20th year, we reflect upon the extraordinary achievements of our students and unwavering support of the academia and community at large. Their efforts and contributions have played a pivotal role in shaping a brighter future through mathematics.

2,600 students

600 teachers

200+ schools

460 research reports

800+ teams

190+ winners

70+ professors
44 universities

HK$10,000,000 prize money

It All Started with an Idea

2004

20 years ago, we envisioned a competition that encourages secondary school students to embark on intellectual discoveries in mathematics and sciences — the first of its kind in Asia.

Co-organized by Hang Lung Properties and CUHK, the first Hang Lung Mathematics Awards (HLMA) attracted over 100 school teams to participate. A good start!

The inaugural HLMA awards ceremony was held at the Government House, with Prof. Yongxiang Lu, then President of the Chinese Academy of Sciences, as the Guest of Honor and over 100 renowned mathematicians around the world in attendance.

Creating a Lasting Legacy

2006

"This [Awards] is a splendid thing and, as far as I know, there really isn't anything like this in the rest of the world."
– Prof. Sir James Mirrlees

HLMA is governed by two principal committees. Prof. Sir James Mirrlees, 1996 Nobel Laureate in Economics, joined the HLMA Steering Committee as Chair, a position he held for 10 years until his retirement in 2016.

Prof. Tony Chan, then at UCLA, chaired the 2006 Scientific Committee. Throughout HLMA history, he has unfailingly supported each HLMA, either as member of Steering Committee or Scientific Committee.

The HLMA Scientific Committee comprises top mathematicians from around the world. In 2006, our stellar line-up included Prof. Jean-Pierre Kahane from the Université Paris-Sud Orsay, Prof. Gilbert Strang from the Massachusetts Institute of Technology, and Prof. John Coates from the University of Cambridge.

HLMA Piques International Interest

2008

Admission Deans from Brown University, Harvard University, and Stanford University travelled to Hong Kong and engaged with HLMA participants at a tea gathering. They also presented awards to our winners.

Dr. Gerald Chan, Non-Executive Director of Hang Lung Properties and co-founder of Morningside, gave a thoughtful speech to our students and guests at the awards dinner.

Our HLMA Steering Committee boasts lifelong educators like Mr. Siu Leung Ma and Dr. Chee Tim Yip, who have served HLMA for 2 decades and are admirable champions for youth development.

Breaking New Ground

2010

Ms. Ching Wong of PLK Centenary Li Shiu Chung Memorial College made history as the first female HLMA Gold Award recipient. Since 2004, 50 female students have made it to the rarefied Oral Defense stage, of which 22 won awards. Go girls!

Heroes among Us

HLMA's success cannot be achieved without the tireless dedication of mathematics teachers, guiding students on their first research endeavors. They are the heroes behind countless success stories.

Ms. Mee Lin Luk of La Salle College and Mr. Chi Keung Lai of Shatin Pui Yng College are HLMA veteran teachers, each having led students into 5 Oral Defenses over a decade.

"It is not the knowledge of the teachers that lead to [their students'] success, but the attitude of always standing behind their students."
– Prof. Thomas Au

Younger mathematics teachers continue to blaze the trail, like Mr. Long Tin Chan of Diocesan Boys' School who led 6 teams into 2 Oral Defenses, and Mr. Ho Fung Lee of Pui Ching Middle School who led 9 teams into 4 Oral Defenses.

Behind the Scenes

2012

All HLMA papers are reviewed in a rigorous multi-step process. Teams that meet the highest academic standards are invited to present their work before the Scientific Committee. It is an intense whole-day marathon, with serious questions and animated discussions. A very exclusive experience for our finalists.

Dare to Dream

Ms. Ewina Pun of St. Mary's College is our first female solo HLMA winner. After graduating from the University of Southern California in Electrical Engineering and Biomedical Engineering, she is now a PhD candidate at Brown University, focusing on brain-machine interface.

The HLMA awards venue is a destination itself – the Asia Society Hong Kong Center is an acclaimed heritage site steeped in historical and architectural significance. Since its opening in 2012, Asia Society has played host to many HLMA ceremonies.

Decade of Excellence

2014

HLMA turns 10! Mrs. Carrie Lam attended 4 HLMAs as Guest of Honor, first as Chief Secretary for Administration and later as Chief Executive of the HKSAR. Her longstanding support to HLMA is exemplary.

HLMA welcomes a very special guest – Prof. Chen-Ning Yang, 1957 Nobel Laureate in Physics. We are very proud to showcase the young talents of Hong Kong and demonstrate how far their dreams can take them.

Passing the Torch

Mr. Alan Fong of Buddhist Sin Tak College and Mr. Yan Lam Fan of Pui Ching Middle School respectively took gold and silver at the 2014 HLMA. Today, they are young mathematics teachers, igniting students' interest to sample the joys of mathematics research.

Local Masters

2016


HLMA attracts not only international mathematicians but also the best mathematicians in Hong Kong, who faithfully come together every 2 years to meet our students. What attracts them is their shared appreciation of our students' hard work, originality, and the audacity to pursue one's dream.

HLMA Goes Social

As part of our broadened efforts to make mathematics more relatable to youth through social media, the "HLMA – To Infinity" Facebook campaign was launched in 2016 and have since garnered multiple PR awards, with 19 million reach to-date.

Happy grins at HLMA after all the sweat and toil. Gold Award winner Mr. Ken Leung of Bishop Hall Jubilee School later graduated from CUHK and is now a PhD candidate at the University of Montréal.

Sharing a Passion for Learning

2018

In May, HLMA proudly released a collection of research papers by HLMA winners, which are distributed free of charge to all schools in Hong Kong, public libraries, and related educational institutions.

"Mathematics is the new English."
– Dr. Owen H. Ko

A Winner Returns to Stage

Dr. Owen H. Ko, one of our earliest HLMA winners, on stage here with Mr. Adriel Chan, Vice Chair of Hang Lung Properties, sharing his personal journey from mathematics to the frontiers of neuroscience. Dr. Ko holds medical degrees from CUHK and a PhD in neuroscience at the University College London. He is currently Assistant Dean (Research) at CUHK Medicine.

A New Chapter

2021

After a one-year hiatus due to the pandemic, a new partnership between Hang Lung Properties and HKUST is formed.

To mark our new partnership, a new crystal trophy was designed, with a visualized representation of an Einstein-Rosen bridge etched inside.

We are honored to have Prof. Richard Schoen, 2017 Wolf Prize Laureate in Mathematics, and Prof. George F. Smoot, 2006 Nobel Laureate in Physics, to chair our Scientific Committee and Steering Committee, respectively.

"The breadth of topics and the depth and clarity of solutions given were very high, certainly at the level of top high-school and beginning years of college students in the U.S."

– Prof. Richard Schoen

Mr. Jay Cheng, 2006 Gold Award winner and currently a District Engineer of the Civil Engineering and Development Department, shared on stage how mathematics can tackle the impacts of climate change.

Restrictions on international travels hindered our overseas Scientific Committee members from flying to Hong Kong. For the first time, the Oral Defense was held in hybrid format over two consecutive days covering five time zones.

Winners Gala

2022

Our first HLMA Winners Gala was held in June, with Prof. Lap-Chee Tsui, President of the Hong Kong Academy of Sciences, delivering Opening Remarks.

Mathematics for Sustainability

A highlight of the evening was a panel discussion where sustainability experts shared their views on how mathematics and big data can help create a better and more sustainable way of life for humanity and the planet.

HKUST launched 2 successful online lectures on machine learning and the evolution of mathematics. The following year, HKUST tailor-made a series of online workshops from how to find a research topic to writing a mathematics research report, which proved to be very popular among HLMA participants and teachers alike.

Hang Lung Properties

Hang Lung Properties Limited (SEHK stock code: 00101) creates compelling spaces that enrich lives. Headquartered in Hong Kong, Hang Lung Properties develops and manages a diversified portfolio of world-class properties in Hong Kong and in the 9 Mainland cities of Shanghai, Shenyang, Jinan, Wuxi, Tianjin, Dalian, Kunming, Wuhan and Hangzhou. With its luxury positioning under the "66" brand, the company's Mainland portfolio has established its leading position as the "Pulse of the City". Hang Lung Properties is recognized for leading the way in enhanced sustainability initiatives in real estate as it pursues sustainable growth by connecting customers and communities.

We Do It Well is our motto, which speaks to the reaffirmation of our core values of integrity, sustainability, excellence, and openness; and our commitment to working towards the common good of all our stakeholders, our people, our communities, and our environment.

We are committed to cultivating the aspirations of the younger generation through a wide range of comprehensive and holistic empowerment initiatives, with the aim of broadening their horizons, and enabling them to make right decisions for their own future.

In 2004, we founded its signature community investment initiative, Hang Lung Mathematics Awards, an innovative platform to nurture talented young minds and encourage secondary school students to realize their creative potential in mathematics and sciences, and stimulating their passion for intellectual discovery.

In 2022, we stepped up our youth development efforts to help build more diverse and inclusive communities. We launched the inaugural edition of the Hang Lung Future Women Leaders Program in Hong Kong and mainland China to unleash the potential and sharpen the competitive edge of young female university students through active engagement with the government, industry, and society to promote and support women's development while enhancing community wellbeing. Around 180 female university students from across the nation have participated and received over 6,300 hours of leadership training and mentorship.

In 2023, we mobilized our employees to take part in the HKSAR government-led mentorship Strive and Rise Programme that aims to facilitate upward mobility for secondary school students from underprivileged families. Our volunteers participated in a one-year mentorship program and interacted with the mentees by sharing their life experiences.

In addition, we are dedicated to fostering the development of fresh graduates through The Hang Lung Management Trainee Program. This 18-month program is designed to equip young talents with essential professional knowledge and skills across business functions as well as with necessary leadership competencies to excel in the dynamic real estate industry in Hong Kong and mainland China.

As part of our vital commitment to the communities in which we operate, the Hang Lung As One Volunteer Team was established in Hong Kong in 2012. Since then, all of our 11 projects across 9 Mainland cities have formed their own teams of local volunteers, actively cooperating with diverse community groups and charitable organizations to support the wellbeing of society. Over a decade of community engagement, over 11,400 volunteers contributed an accumulated 135,000 service hours to local communities.

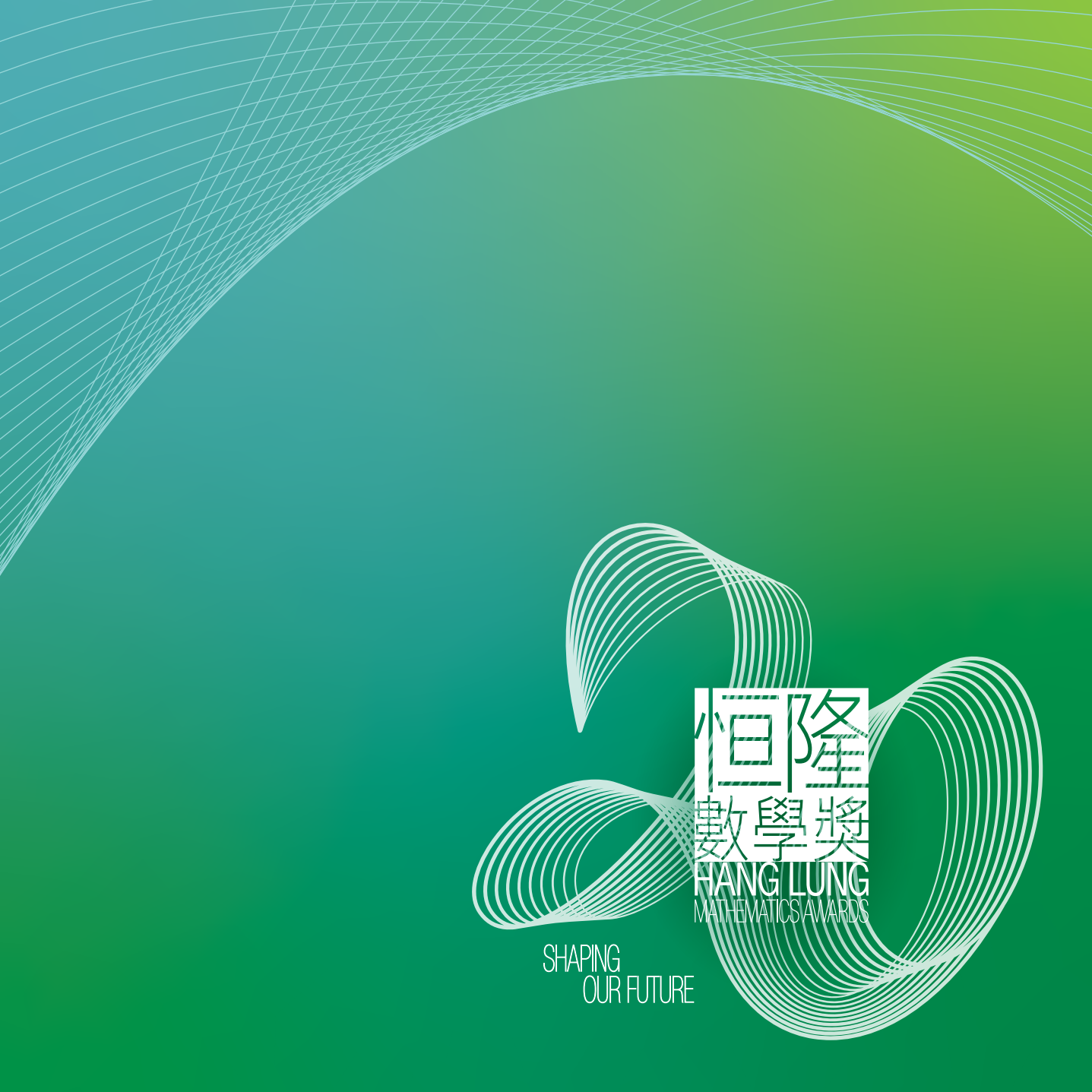
The Hong Kong University of Science and Technology

The Hong Kong University of Science and Technology (HKUST) (https://hkust.edu.hk/) is a world-class research intensive university that focuses on science, technology and business as well as humanities and social science. HKUST offers an international campus, and a holistic and interdisciplinary pedagogy to nurture well-rounded graduates with global vision, a strong entrepreneurial spirit and innovative thinking. Over 80% of our research work were rated “Internationally excellent” or “world leading” in the Research Assessment Exercise 2020 of Hong Kong’s University Grants Committee.

We were ranked 2nd in Times Higher Education’s Young University Rankings 2023, and our graduates were ranked 30th worldwide and among the best from universities from Asia in Global Employability University Ranking and Survey 2022. As of September 2023, HKUST members have founded 1,747 active start-ups, including 9 Unicorns and 13 exits (IPO or M&A), generating economic impact worth over HK$ 400 billion.

InvestHK cited QS World University Rankings by Subject 2021 to demonstrate the performance of five world’s top 100 local universities in several innovation-centric areas, among which HKUST ranked top in four engineering and materials science subjects.

The School of Science and the Department of Mathematics are committed to pursuing cutting-edge research, making groundbreaking discoveries and establishing new research paradigms. Our quality and well-balanced education places particular emphasis on grit, curiosity, and creativity. We are dedicated to equipping our students with the knowledge and confidence to be inspirational leaders who are capable of making a difference in society.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MATHEMATICS AWARDS
SHAPING
OUR FUTURE