



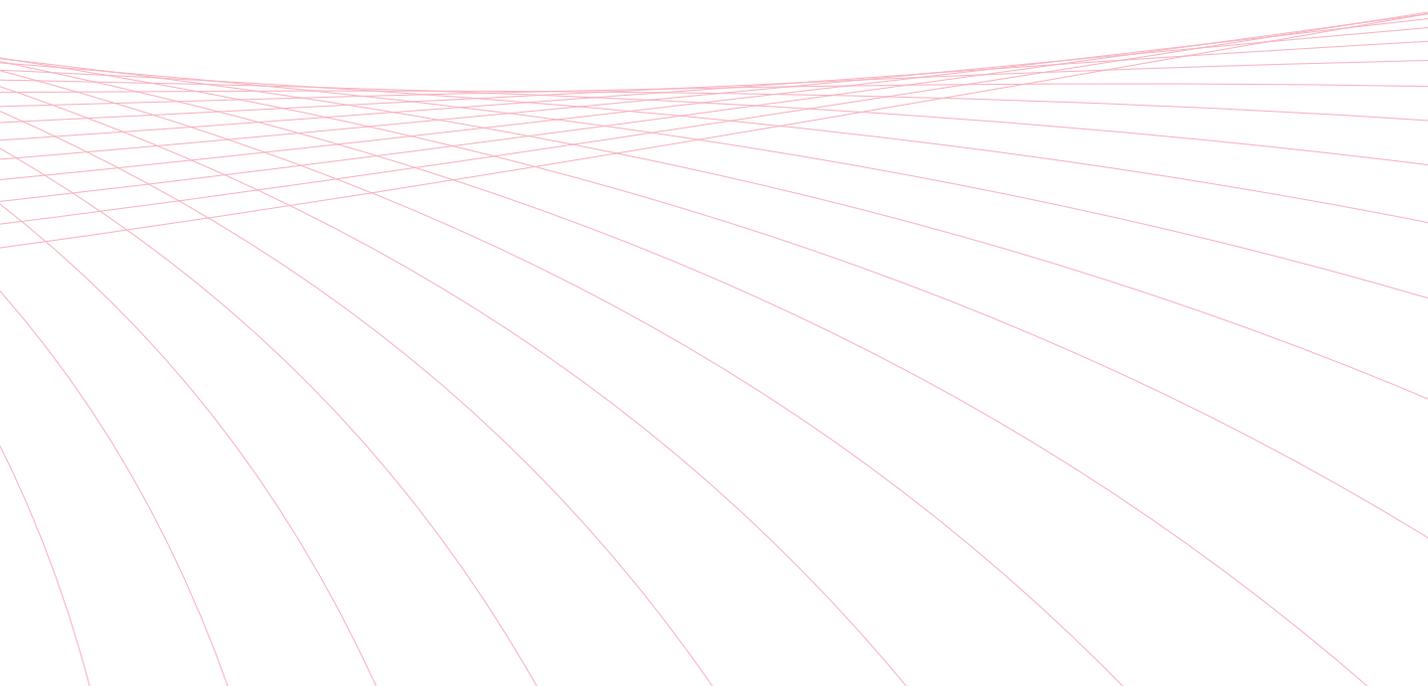
恒隆  
數學獎  
HANG LUNG  
MATHEMATICS AWARDS





## **2021 Hang Lung Mathematics Awards Announcement and Awards Presentation**

December 15, 2021  
Conrad Hong Kong  
Pacific Place, 88 Queensway, Hong Kong



# Program

## Welcome Remarks

Mr. Ronnie C. Chan

Chair  
Hang Lung Properties Limited

Professor Wei Shyy

President  
The Hong Kong University of Science and Technology

Professor Richard Schoen

Chair, Scientific Committee  
2021 Hang Lung Mathematics Awards

Professor George F. Smoot

Chair, Steering Committee  
2021 Hang Lung Mathematics Awards

## Dinner

## Opening Remarks

The Hon. Mrs. Carrie Lam  
Cheng Yuet-ngor, GBM, GBS

The Chief Executive  
Hong Kong Special Administrative Region

## Dialog: How Mathematics can Tackle the Impacts of Climate Change

Mr. Jay Cheng

District Engineer  
Civil Engineering and Development Department  
The Government of the Hong Kong Special Administrative Region

Mr. Adriel Chan (moderator)

Vice Chair  
Hang Lung Properties Limited

## Vote of Thanks and Presentation of Mementos

Mr. Weber Lo

Chief Executive Officer  
Hang Lung Properties Limited

## 2021 Hang Lung Mathematics Awards Announcement and Awards Presentation

## Group Photos

## Welcome Letter

Dear Friends,

2021 has been another extraordinarily challenging year for our society, economy, and, certainly, education. We are pleased that Hang Lung Mathematics Awards (HLMA) continues to flourish as it enters into its next phase of development. This year marks a new partnership between Hang Lung Properties and The Hong Kong University of Science and Technology, a young but dynamic university with focus on science, technology, and business, and a pioneer in interdisciplinary pedagogy. We look forward to working together and forge an exciting new era of innovation and engagement.

What started as an idea 17 years ago came to fruition, and the meticulous execution of the dedicated professors at The Chinese University of Hong Kong (CUHK) over the ensuing years have made HLMA an enviable success. We are glad that our inaugural partner CUHK continues to work closely with us on this meaningful project, to empower Hong Kong's secondary school students to realize their creative potential in mathematics and the sciences and ignite their passion for intellectual discovery.

This year, under the new partnership, we are honored to have Professor Richard Schoen, 2017 Wolf Prize Laureate in Mathematics, to serve as Chair of the Scientific Committee, and Professor George F. Smoot, 2006 Nobel Laureate in Physics, to chair our Steering Committee. The two principal committees boast distinguished mathematicians and educators from around the globe. Our gratitude goes to the long serving members for their continuous support, some of whom have been associated with us since inauguration, and we are also pleased to welcome new members who would no doubt bring new perspectives and energy.

Over the years, the enthusiastic participation of local students and schools, the caliber of the research papers, and the reputation of this competition in the academic community and in society are testaments to the success of the Awards. In this spirit, we look forward to celebrating the hard work and accomplishments of our students this evening. We wish them all the best in their future intellectual journeys.



**Mr. Ronnie C. Chan**  
Chair  
Hang Lung Properties Limited



**Professor Wei Shyy**  
President  
The Hong Kong University of Science and Technology

## Welcome Letter

Dear Students, Teachers, and Honored Guests,

Welcome to the Announcement and Awards Presentation of the 2021 Hang Lung Mathematics Awards (HLMA). As the pandemic situation in Hong Kong continued to improve in 2021, and social distancing measures have been relaxed in recent months, we felt this is an auspicious time to come together and celebrate the hard work and achievements of our students in person. The Honorable Mrs. Carrie Lam Cheng Yuet-ngor, The Chief Executive of the Hong Kong Special Administrative Region, has been a great supporter and friend of HLMA for many years, and we are very honored to have her deliver the opening address this evening.

This year marks a new partnership between Hang Lung Properties and The Hong Kong University of Science and Technology. To provide guidance and support to our participating students, the Mathematics Department of The Hong Kong University of Science and Technology hosted a series of online workshops throughout 2021, with topics ranging from research techniques and how to write a good research report, to specialized mathematics topics such as abstraction in modern mathematics and research in applied and computational mathematics. These online workshops were well received, and we are pleased to see a strong turnout of research reports submitted.

Restrictions on international travel have hindered our Scientific Committee and Steering Committee members based overseas from flying to Hong Kong to participate in the competition and join us this evening in person. For the first time, we have held our Oral Defense in hybrid format over two consecutive days covering five time zones. Our thanks go to all those who have tirelessly worked to ensure the competition can be run smoothly.

As we embrace many changes, we are also delighted to savor some traditions, like featuring an HLMA alum at the awards presentation. Tonight, we are glad to have Mr. Jay Cheng, 2006 HLMA Gold Award winner and currently a District Engineer of the Civil Engineering and Development Department of the Hong Kong Government, to join us and share how mathematics can tackle the impacts of climate change.

Our thanks go to the Scientific Committee, Steering Committee, Executive Committee, and Screening Panel members for their invaluable guidance and dedication in making HLMA a success. We also like to express our heartfelt appreciation to the students, teachers, and schools for your support and encouragement.

In appreciation,



**Mr. Weber Lo**  
Chief Executive Officer  
Hang Lung Properties Limited



**Professor Richard Schoen**  
Chair, Scientific Committee  
2021 Hang Lung Mathematics Awards



**Professor George F. Smoot**  
Chair, Steering Committee  
2021 Hang Lung Mathematics Awards



**Professor Yang Wang**  
Vice-President for Institutional Advancement  
The Hong Kong University of Science and Technology

## Hang Lung Mathematics Awards

Hang Lung Mathematics Awards is a highly regarded biennial mathematics research competition for secondary school students in Hong Kong. Founded in 2004, Hang Lung Mathematics Awards blazed a trail in secondary school mathematics education, encouraging students to realize their creative potential in mathematics and sciences, and stimulate their passion for intellectual discovery.

Since the inaugural awards in 2004, Hang Lung Properties, in partnership with The Institute of Mathematical Sciences and the Department of Mathematics of The Chinese University of Hong Kong, has successfully run eight editions of Hang Lung Mathematics Awards. Over 2,400 students from 200 schools have participated in the competition, and over 400 research papers were submitted. Hang Lung Mathematics Awards has made a profound impact in nurturing a pool of young mathematics talents in Hong Kong. Among our earlier winners, many have completed their studies from top universities around the world and are now working in various sectors, such as academia and other professions, and making meaningful contributions to the society. A special note of thanks goes to our inaugural partner, The Chinese University of Hong Kong, for their vision, guidance, dedication, and support they have given to Hang Lung Mathematics Awards' success.

In 2021, Hang Lung Properties formed a new partnership with The Hong Kong University of Science and Technology. Ranked as one of the world's top young universities, The Hong Kong University of Science and Technology is dedicated to the nurture of young talents, encouraging them to discover their creativity and innovativeness and to pursue entrepreneurship. Under this new partnership, Hang Lung Properties donates HK\$2.5 million to each competition, of which HK\$1 million is set aside as monetary prizes, and the remainder goes towards academic consultancy, assessment process, the administration of the competition, as well as education promotion activities. The Hong Kong University of Science and Technology handles the academic and educational aspects of the competition. In addition, the university offers scholarships for teachers nominated by the winning schools to enroll in the Master of Science Program in Mathematics for Educators to recognize and encourage teachers for their contributions towards mathematics education in Hong Kong.

Hang Lung Mathematics Awards has two principal committees, the Scientific Committee, the academic and adjudicating body of the competition, and the Steering Committee, an advisory body that provides support, guidance, and oversight of the Awards. Professor Richard Schoen, 2017 Wolf Prize Laureate in Mathematics, is the Chair of the 2021 Hang Lung Mathematics Awards Scientific Committee, which boasts 14 renowned mathematicians from eminent universities around the globe. Professor George F. Smoot, 2006 Nobel Laureate in Physics, chairs the 2021 Hang Lung Mathematics Awards Steering Committee, which comprises mathematicians, scientists, and leaders from different sectors of society.

## Hang Lung Mathematics Awards

Schools are invited to form teams of up to five students. Under the supervision of a teacher, each team decides on a mathematics topic, designs and carries out a mathematics research project, then submits a research report that summarizes the methodology, research, and results. A Scientific Committee will evaluate the research reports in a rigorous, multi-step review process that is comparable to publishing an article in a scientific journal, and determine the teams that will be invited to participate in an oral defense. The oral defense is modeled after the doctoral dissertation defense and comprises a brief public presentation of the research project, followed by a closed-door inquiry by the Scientific Committee. At the conclusion of the oral defense, the Scientific Committee will decide the winners of the Hang Lung Mathematics Awards, and the results will be announced at the awards presentation ceremony.

Over 60 teams from nearly 40 secondary schools in Hong Kong have participated in the 2021 Hang Lung Mathematics Awards. At each Hang Lung Mathematics Awards, up to eight awards will be presented to recognize those mathematics research projects that meet the highest academic standards in terms of methodology, research, and scholarship. The eight awards are: a Gold Award, a Silver Award, a Bronze Award, and up to five Honorable Mentions.

Each award has four components: a Student Education Award, to be shared equally among team members and applied towards their university studies; a Teacher Leadership Award, for the supervising teacher; a School Development Award, for the winning school to promote mathematics education, and an Educator Scholarship, for a teacher nominated by the winning school. Besides monetary prizes, each winning student and teacher will receive a crystal trophy and a certificate. Each winning school will receive a crystal trophy.

Etched inside the crystal trophy is a visualized representation of an Einstein-Rosen bridge. An Einstein-Rosen bridge, sometimes called a "wormhole", is a transcendental bijection of the spacetime continuum, an asymptotic projection of the Calabi-Yau manifold manifesting itself in Anti-de Sitter space, first described by Ludwig Flamm in 1916 and rediscovered by Albert Einstein and Nathan Rosen in 1935. It represents the geometry of a "throat" that joins disparate regions of spacetime, but it is non-traversable, i.e., no physical timelike trajectory crosses the bridge. It plays a prominent role in mathematics and physics, where it has implications for geometry, gravity, information, and quantum physics.

## Hang Lung Mathematics Awards

in HK\$	Student Education Award	Teacher Leadership Award	School Development Award	Educator Scholarship	Amount of each Award
1x Gold Award	\$250,000	\$50,000	\$100,000	Full program fee	\$400,000 + Educator Scholarship
1x Silver Award	\$120,000	\$20,000	\$60,000	Full program fee	\$200,000 + Educator Scholarship
1x Bronze Award	\$60,000	\$10,000	\$30,000	Full program fee	\$100,000 + Educator Scholarship
5x Honorable Mentions	\$32,000	\$8,000	\$20,000	Half program fee	\$60,000 + Educator Scholarship
Total	<b>HK\$1,000,000 + Educator Scholarships</b>				



# 2021 Hang Lung Mathematics Awards Committees and Panel

## SCIENTIFIC COMMITTEE

### Chair



**Professor Richard Schoen**  
2017 Wolf Prize Laureate in Mathematics  
Stanford University  
University of California, Irvine

### Members



**Professor Raymond Hon Fu Chan**  
City University of Hong Kong



**Professor Jun Kigami**  
Kyoto University



**Professor Andrew G. Cohen**  
The Hong Kong University of  
Science and Technology



**Professor Gitta Kutyniok**  
Ludwig Maximilian University of Munich



**Professor Ingrid Daubechies**  
Duke University



**Professor Wei-Ping Li**  
The Hong Kong University of  
Science and Technology



**Professor Frederick Tsz-Ho Fong**  
The Hong Kong University of  
Science and Technology



**Professor Ngaiming Mok**  
The University of Hong Kong



**Professor Jill Pipher**  
Brown University



**Professor Van Vu**  
Yale University



**Professor Tom Yau Heng Wan**  
The Chinese University of Hong Kong



**Professor Ya-xiang Yuan**  
Chinese Academy of Sciences



**Professor Shou-Wu Zhang**  
Princeton University

# 2021 Hang Lung Mathematics Awards Committees and Panel

## STEERING COMMITTEE

### Chair



**Professor George F. Smoot**

2006 Nobel Laureate in Physics  
The Hong Kong University of Science and Technology

### Members



**Professor Thomas Kwok Keung Au**

The Chinese University of Hong Kong



**Professor Shing Yu Leung**

The Hong Kong University of  
Science and Technology



**Professor Tony F. Chan, JP**

King Abdullah University of  
Science and Technology



**Mr. Siu Leung Ma, BBS, MH**

Fung Kai Public School



**Mr. Alfred Cheung**

Tianda Institute



**Professor Michael Kwok Po Ng**

The University of Hong Kong



**Dr. Owen H. Ko**

The Chinese University of Hong Kong



**Professor Xue-Cheng Tai**

Hong Kong Baptist University



**Professor Yang Wang**  
The Hong Kong University of  
Science and Technology



**Ms. Susan Wong**  
Hang Lung Properties Limited



**Dr. Chee Tim Yip**  
Princeton (Shenzhen) International School



**Professor Ding-Xuan Zhou**  
City University of Hong Kong

# 2021 Hang Lung Mathematics Awards Committees and Panel

## SCREENING PANEL

### Chair



**Professor Frederick Tsz-Ho Fong**

The Hong Kong University of  
Science and Technology

### Members



**Professor Ivan Chi Ho Ip**

The Hong Kong University of  
Science and Technology



**Professor Tianling Jin**

The Hong Kong University of  
Science and Technology



**Dr. Albert Yin Bon Ku**

The Hong Kong University of  
Science and Technology



**Professor Eric Marberg**

The Hong Kong University of  
Science and Technology



**Professor Maosheng Xiong**

The Hong Kong University of  
Science and Technology

## EXECUTIVE COMMITTEE

### Chair



**Professor Shing Yu Leung**

The Hong Kong University of  
Science and Technology

### Members



**Professor Jimmy Chi Hung Fung**

The Hong Kong University of  
Science and Technology



**Dr. Tsz Kin Lam**

The Hong Kong University of  
Science and Technology



**Dr. Chi Man Leung**

The Hong Kong University of  
Science and Technology



**Dr. Chi Wai Yu**

The Hong Kong University of  
Science and Technology

### Secretariat



**Ms. Niki Hiu Yan Chan**

The Hong Kong University of  
Science and Technology

## A Tribute to Professor Ka-Sing Lau

Hang Lung Mathematics Awards (HLMA) pays tribute to Professor Ka-Sing Lau (1948-2021), who played a key role in the founding of HLMA, and was one of our longest serving Scientific Committee members.

Professor Lau was Chairman of the Department of Mathematics at The Chinese University of Hong Kong when HLMA was founded in 2004. He secured the support of the whole department and encouraged many faculty members to help organize this unique mathematics research competition for secondary school students, the first of its kind in Asia.

Professor Lau attended our HLMA preparation meetings and served on our Scientific Committee from 2004 to 2010, and continued to serve on the Scientific Committee and Steering Committee after his retirement from 2012 to 2016. He solicited many external referees and very distinguished mathematicians for HLMA, and paid much attention and effort to ensure the sustainability of HLMA. His longstanding support to HLMA provided fertile ground upon which HLMA flourished and grew from strength to strength.

We are deeply saddened by Professor Lau's passing, and would like to express our sincere gratitude to Professor Lau for his invaluable guidance and advice to HLMA. It had been our great pleasure and honor to work with him on this meaningful journey in nurturing young mathematics talents in Hong Kong.



## 2021 Hang Lung Mathematics Awards

### Finalists (listed by school name in alphabetical order)

- School : Diocesan Boys' School  
Teacher : Mr. Long Tin Chan  
Team Member : Myron Lam  
Topic : An Investigation on the Rings of Integer-Valued Polynomials on Gaussian Integers and Integer-Valued Continuous Functions
- School : Diocesan Boys' School  
Teacher : Mr. Long Tin Chan  
Team Member : Tsz Fung Chui  
Topic : On the Basel Problem: Generalizations to Other Power Series
- School : Diocesan Boys' School  
Teacher : Mr. Long Tin Chan  
Team Members : Nick Hang Cheng, Hei Chai Shum, Chun Shing Tong  
Topic : On the Variations of the Brachistochrone Curve
- School : HKUGA College  
Teacher : Mr. Yuk Lun Fong  
Team Members : Yan Yin Cheung, Yu Wong  
Topic : Reinforcement Learning on Geometric Construction
- School : Kwun Tong Government Secondary School  
Teacher : Mr. Ka Fai Ip  
Team Member : Wai Yang  
Topic : On Polyadic Group and Ordinary Polyadic Equation
- School : Kwun Tong Maryknoll College  
Teacher : Mr. Ka Ming Chan  
Team Members : Yuk Hang Kwok, Tsz Shing Li  
Topic : Investigation on Riemann Zeta Function and P-adic Number
- School : Pui Ching Middle School  
Teacher : Mr. Ho Fung Lee  
Team Member : Tsz Hin Chan  
Topic : On Non-Torsion Solutions of Homogeneous Linear Systems over Rings
- School : Pui Ching Middle School  
Teacher : Mr. Ho Fung Lee  
Team Member : On Ki Luo  
Topic : The Johnson-Leader-Russell Question on Square Posets

9. School : S.K.H. Lam Woo Memorial  
Secondary School  
Teacher : Mr. Cathay Kwok Tai Wong  
Team Members : Hoi Nam Cheung, Hok Ip Chow,  
Hin To Chu, Chi Kin Yau  
Topic : From Happy Function to Linear  
Happy Functions
10. School : S.K.H. Tsang Shiu Tim Secondary School  
Teacher : Mr. Ka On Cho  
Team Members : Chun Hei Mok, Chi Kong Wei, Wang Kin To  
Topic : On the Generalization of the Triangle Peg  
Problem to Higher Dimensions
11. School : St. Paul's Co-educational College  
Teacher : Mr. Pak Leong Cheung  
Team Member : Wai Lok Lai  
Topic : Solvability of the General Pell's Equation,  
Quadratic Residuosity, and Real Quadratic  
Fields of Class Number Two
12. School : St. Paul's Convent School  
Teacher : Dr. Chun Yin Dy  
Team Members : Yongzhen Jiang, Yu Sing Keung, Tsz Hin Lui  
Topic : On the Moving Sofa Problem
13. School : TWGHs Lo Kon Ting Memorial College  
Teacher : Ms. Ching Chi Lam  
Team Member : Yuk Ming Chan  
Topic : Integrality of Generalized Binomial  
Coefficient
14. School : Wah Yan College, Kowloon  
Teacher : Mr. Wai Man Chu  
Team Members : Hui Yat Cheung, Pak Kiu Fong,  
Shing Him Lee, Justin Ka Chai Leung,  
Siu Lam Ting  
Topic : Investigation on the Buffon-Laplace  
Needle Problem

## 2021 Hang Lung Mathematics Awards

### Finalists' Abstracts (listed by school name in alphabetical order)

#### 1. Diocesan Boys' School

##### **An Investigation on the Rings of Integer-Valued Polynomials on Gaussian Integers and Integer-Valued Continuous Functions**

This is an investigation on the ring of integer-valued polynomials on the Gaussian integers and the ring of integer-valued continuous function on rational integers, inspired by the results from integer-valued polynomials on the rational integers. Polynomials in the first ring map Gaussian integers to Gaussian integer values while functions in the second ring map rational integers to rational integers. This investigation explores their properties as rings, following a chain of class inclusions, which includes the most commonly known domains. The properties of rings of polynomials over algebraic integers, continuously differentiable functions on rational integers, and continuous functions on Gaussian integers are also discussed.

#### 2. Diocesan Boys' School

##### **On the Basel Problem: Generalizations to Other Power Series**

The Basel problem is about finding the sum of the reciprocals of all perfect squares. This problem is first posed by Pietro Mengoli in 1650 and was solved by Leonhard Euler in 1734. Euler proved that the sum of the series is  $\pi^2/6$ . In this report, inspired by an idea suggested by the YouTube channel 3blue1brown in 2018, we attempt to give a new proof to the Basel problem. After that, we discuss some possible generalizations of the Basel problem, by finding the sum of reciprocals of squares and cubes of the form  $an+b$ . Furthermore, we discuss how the sum of reciprocals of integral powers of  $an+b$  can be computed, and the relation between  $\zeta(3)$  and the results we have achieved.

#### 3. Diocesan Boys' School

##### **On the Variations of the Brachistochrone Curve**

In this research report we will provide our own derivation of the classical Brachistochrone Curve. Afterwards, we will study some variations of the Brachistochrone Problem with two gravitational forces, friction, and finally some concepts from non-Newtonian mechanics.

#### 4. HKUGA College

##### **Reinforcement Learning on Geometric Construction**

This paper aims to discuss the procedure of applying reinforcement learning in geometric construction. The significant achievement of this paper includes a relatively general method of finding sufficient points for completing designated construction tasks, an algorithm suggesting the selection of necessary points in the process of construction, and two reinforcement learning models. The first learning model using greedy algorithms generates efficient construction results of four tasks specifically chosen from Euclidea. Moreover, the first model also generates a construction method of the regular heptadecagon. The second reinforcement learning model involves deep Q-learning. The framework and training demonstration are given, yet by all kinds of limitations, only an intermediate result is achieved. Eventually, the paper hopes to build connections between machine learning and the development of mathematics discovery.

#### 5. Kwun Tong Government Secondary School

##### **On Polyadic Group and Ordinary Polyadic Equation**

The main aims of this paper are to give an insight on the binary algebraic structure called polyadic group (or  $n$ -ary group) and provide an initial study of an equation called polyadic equation, especially the ordinary polyadic equation (OPE). In this paper, we investigated  $n$ -ary group and related terms, for example: ternary group and the Hosszú-Gluskin theorem. Besides that, we established a classification of polyadic equations. After that, we studied ordinary polyadic equations in different forms.

6. Kwun Tong Maryknoll College

### **Investigation on Riemann Zeta Function and P-adic Number**

In this paper, we will study the connection between the two concepts that are the Riemann zeta function  $\zeta(s)$  and the p-adic number system. We have found that there is a p-adic analogue of zeta function,  $\zeta_p(s)$ , and its generalization, the p-adic L-function  $L_p(s, \chi)$ , is very important to modern mathematics such as the studies of elliptic curves and modular forms.

7. Pui Ching Middle School

### **On Non-Torsion Solutions of Homogeneous Linear Systems over Rings**

In this paper, we study the existence of non-torsion solutions of a homogeneous linear system over a commutative ring. More precisely, we determine the minimal positive integer  $n$  such that any homogeneous systems of  $m$  equations with  $n$  variables over a given ring  $R$  gives a non-torsion solution, i.e. a solution  $x = (x_1, x_2, \dots, x_n)$  such that at least one coordinate  $x_i$  is not a zero-divisor. We proved that over Noetherian rings, a non-trivial lower bound to the minimal number can be guaranteed via the use of primary decomposition. We also consider the number of generators of ideals in  $R$  and the localisations of  $R$ . For some classes of Noetherian rings, such as principal ideal rings and reduced rings, we show that such minimal number exists.

8. Pui Ching Middle School

### **The Johnson-Leader-Russell Question on Square Posets**

We study the problem of finding the maximum number of maximal chains in a given size- $k$  subset of a square poset  $[n] \times [n]$ . This was proposed by Johnson, Leader, and Russell but not yet solved. Kittipassorn had given a conjectural solution to the problem. We verify Kittipassorn's conjecture for  $0 \leq k \leq 3n-2$  and solve a variant problem for the case  $3n-1 \leq k \leq 4n-4$ , which also supports the conjecture. For general  $k$ , we find that the optimal configuration is given by a 1-Lipschitz function. We also generalize the problem to rectangle posets and give a solution to one particular poset.

9. S.K.H. Lam Woo Memorial Secondary School

### **From Happy Function to Linear Happy Functions**

Given  $p, q \in \mathbb{N}$  and  $b \in \mathbb{N} \setminus \{1\}$ , the linear happy function is defined as  $H(x) = ph(x) - q$ , where  $h(x)$  is the 'happy function' that maps a positive integer to the squares of its base- $b$  digits. When there exists a positive integer  $m$  such that the  $m$ -th iterate of  $H(x)$  on  $x$  is 1, i.e.  $H_m(x) = 1$ , then  $x$  is called a 'happy number' in the linear happy function. In this paper, we found the necessary and sufficient conditions for  $p$  and  $q$  so that happy number exists. We have also investigated the existence of different forms of happy numbers, and found a formula to find  $L$  such that  $H(x) < x$  for all  $x \geq L$ .

10. S.K.H. Tsang Shiu Tim Secondary School

### **On the Generalization of the Triangle Peg Problem to Higher Dimensions**

A lot of work has been put into solving special cases of the famous Square Peg Problem, which focuses on the two-dimensional space. The aim of this project is to investigate the generalization of the Triangle Peg Problem into manifolds of higher dimensions.

By considering the Triangle Peg Problem, we have successfully proven the Tetrahedron Peg Problem, i.e. for every smooth compact connected surface, there exists four distinct points on the surface which can form a regular tetrahedron. Finally, by induction, we have generalized a variant of the Triangle Peg Problem to even higher dimensions.

11. St. Paul's Co-educational College

### **Solvability of the General Pell's Equation, Quadratic Residuosity, and Real Quadratic Fields of Class Number Two**

A new and practical test for determining the solvability of the general Pell's equation  $x^2 - Dy^2 = n$  will be offered through proving one necessary condition and one sufficient condition for this renowned quadratic Diophantine equation to be solvable in integers. The test involves only prime factorization and checking of certain simple quadratic residuosity relations. While the necessary condition will be comparatively more straightforward, the sufficient condition in a form of conditional converse of the former will require algebraic number theory tools to formulate and analyze. To prove this sufficient condition, the solvability in question will be transformed into the question of principality of certain well-designed ideal class in a real quadratic field  $Q(\sqrt{D})$  of class number two.

12. St. Paul's Convent School  
**On the Moving Sofa Problem**

The Moving Sofa Problem asks for the maximal area of a two-dimensional shape that can be moved around a right-angled corner in a hallway of unit width. We represent the sofa boundary as a parametrized curve to find the integrals that the area-maximised sofa must satisfy. Then, after applying calculus of variations to said integrals to maximise them, we use Green's theorem to find the equations of the sofa boundary. We also present a computational approach which is able to derive the best shape and area for generalised cases of the problem of non-right-angled hallways.

13. TWGHs Lo Kon Ting Memorial College  
**Integrality of Generalized Binomial Coefficient**

We investigate the integrality of one possible generalization of the factorial formula of the well-known binomial coefficients, in which the sum of integers in the denominator is greater than the numerator. We first prove a lower bound for the number of these integral fractions for a given numerator. We then discover one particular pattern of them that is related with the base  $p$ -expansion of integers. Finally, we give some corollaries and a graphical representation of them.

14. Wah Yan College, Kowloon  
**Investigation on the Buffon-Laplace Needle Problem**

The Buffon-Laplace needle problem is a variation of the well-known Buffon's needle problem, which asks what the probability of a needle, after being dropped to a rectangular grid, intersects, or touches the grid is. In this paper, we aim to solve some generalizations of this problem. We generalized the problem by dropping regular polygons to the grid instead of dropping a needle. We solved this generalization of the problem by first using the rotational and reflectional symmetry of the regular polygons, then splitting the number of sides of the polygon into 4 cases, then solving each case. We also generalized the problem by dropping arbitrary 2D shapes. We found a general formula and an algorithmic solution to the problem. Apart from generalizations to the problem, we also considered some variations of the problem, like dropping right regular polygon prisms in a 3D space, with the grid being planes in each axis. We used a similar method to solve this problem and provided a formula. We also considered dropping a needle into a  $n$ -dimensional space. However, we failed to get a closed form for the formula.

## Hang Lung Properties

Hang Lung Properties Limited (stock code: 00101) creates compelling spaces that enrich lives. Headquartered in Hong Kong, Hang Lung Properties develops and manages a diversified portfolio of world-class properties in Hong Kong and nine Mainland cities: Shanghai, Shenyang, Jinan, Wuxi, Tianjin, Dalian, Kunming, Wuhan, and Hangzhou. With its luxury positioning under the “66” brand, the company’s Mainland portfolio has established a leading position as the “Pulse of the City”. Hang Lung Properties is recognized for leading the way in enhanced sustainability initiatives in real estate as it pursues sustainable growth by connecting customers and communities.

**We Do It Well** is our motto and speaks to our reaffirmation of our core values of integrity, sustainability, excellence, and openness; and our commitment to working towards the common good of all our stakeholders, our people, our communities, and our environment.

Hang Lung has a long tradition of supporting education. In 2004, the Company founded its signature community investment initiative, Hang Lung Mathematics Awards, an innovative platform to nurture talented young minds and showcase their mathematical and scientific creativity.

In 2017 and 2019, Hang Lung organized the year-long “Hang Lung Young Architects Program”, through which students were able to explore the relationship between architecture and the community, and hone their own architectural knowledge and skills.

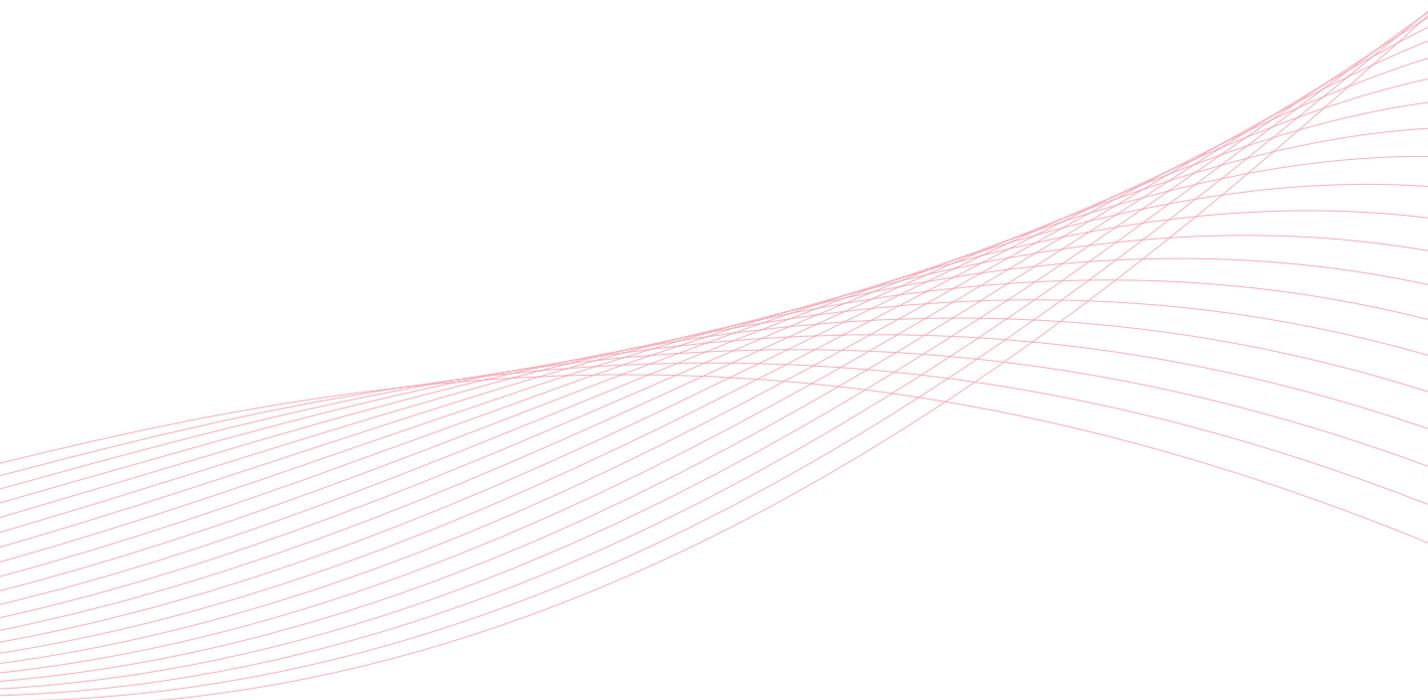
As part of our commitment to the communities in which we operate, the Hang Lung **As One Volunteer Team** was established in 2012 in Hong Kong, and all of our 11 projects across nine Mainland cities now have their own teams of local volunteers. In 2020 alone, our volunteer teams organized approximately 100 volunteer activities in Hong Kong and on the Mainland, together contributing 13,000 hours of voluntary service to local communities, with a focus on diversity and inclusion, youth development and education, helping the elderly, and environmental protection.

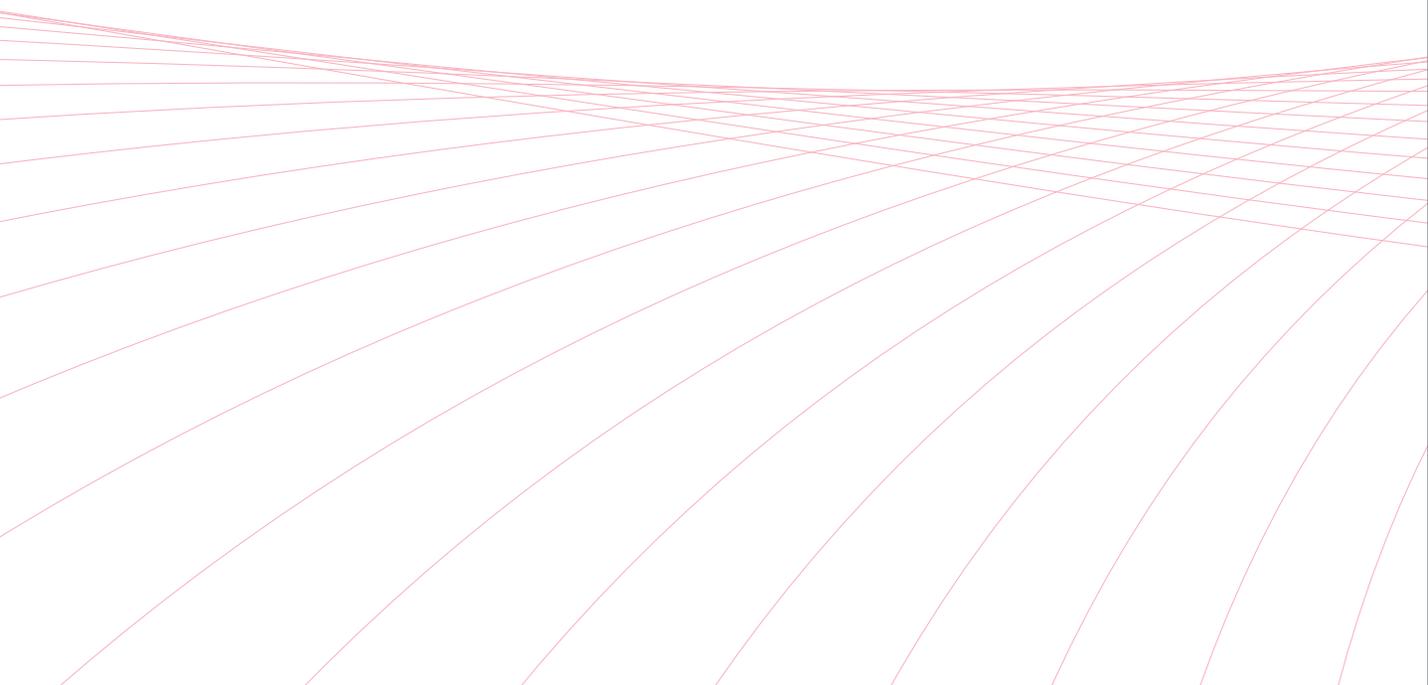
## The Hong Kong University of Science and Technology

The Hong Kong University of Science and Technology (HKUST) is a world-class research university that focuses on science, technology, business and management, as well as humanities and social sciences. We offer an international campus, and a holistic and interdisciplinary pedagogy to nurture well-rounded graduates with a global vision, a strong entrepreneurial spirit, and innovative thinking.

As a research university, innovation lies at the heart of who we are, drawing on HKUST's inner drive and passion to advance the future. This continuous momentum is born of our core belief that knowledge empowers and research delivers, leading to novel discoveries and productive social change. Within a short history, HKUST has already established a strong presence in global higher education and received a number of international recognitions. It has rapidly risen in the global academic arena, ranking among the top-tier universities in Asia.

The School of Science and the Department of Mathematics are committed to pursuing cutting-edge research, making groundbreaking discoveries and establishing new research paradigms. Our quality and well-balanced education places particular emphasis on grit, curiosity, and creativity. We are dedicated to equipping our students with the knowledge and confidence to be inspirational leaders who are capable of making a difference in society.









恒隆  
數學獎  
HANG LUNG  
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