DECONSTRUCTING KAFKA

TEAM MEMBERS

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ABSTRACT. The relationship between Kafka and his father was modelled by a system of differential equations. The results were compared with the real situation. A brief conclusion was drawn concerning the factors that may influence the relationship.

1. Introduction

Since calculus was invented by Newton and Leibniz, it has been intensively used to analyse various scientific issues, especially in physics. Differential equations are now the main tools for scientists to accomplish tasks like weather forecasting, nuclear fission control, or population expectation. However, problems on relationships between humans are seldom tackled by this mathematical tool, with an exception of a one-page paper by Strogatz [3] trying to analyse the relationship between Romeo and Juliet.

Our project is motivated by an article《山水之間—四分一》, which is believed to be a simplified version of an article of Sergio Rinaldi [4]. However, both of them concern the love between a man and a woman. In this paper, we would try to extend their methodology (i.e. using ordinary differential equations) to describe the dynamics of love between a father and a son. Moreover, we believe that this analysis should be substantially more complex than the two articles mentioned above. The reason is that besides their personalities, we also need to consider other factors, for example, the growth of the son, the natural parental care, the Oedipus Complex, etc. Our goal would be to take considerations of any major factors possibly affecting their relationship, plot a curve, and then see whether our model is successful

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[1]This work is done under the supervision of the authors' teacher, Mr. Lap-Foo Chu.

or not. If it turns out to be successful, we will use the results of this paper and Rinaldi's [4] to generalize the model.

For this, the famous case of Franz Kafka and his father is chosen. Franz Kafka is considered the greatest prophet in the 20th century, but his relationship with his father had always been awful, which was reflected in lots of his works. In this paper, we seek to find out a few equations governing their complicated relationships, and compare this with the data found by analysing excerpts taken from his stories.

2. Graph by Data

Franz Kafka, b. Prague, Bohemia, July 3, 1883, d. June 3, 1924, has come to be one of the most influential writers of this century. Virtually unknown during his lifetime, the works of Kafka have since been recognized as symbolizing modern man's anxiety-ridden and grotesque alienation in an unintelligible, hostile, or indifferent world. Kafka came from a middle-class Jewish family and grew up in the shadow of his domineering shopkeeper father, who impressed Kafka as an awesome patriarch. The feeling of impotence, even in his rebellion, was a syndrome that became a pervasive theme in his fiction. Kafka did well in the prestigious German high school in Prague and went on to receive a law degree in 1906. This allowed him to secure a livelihood that gave him time for writing, which he regarded as the essence-both blessing and curse-of his life. He soon found a position in the semipublic Workers' Accident Insurance institution, where he remained a loyal and successful employee until-beginning in 1917-tuberculosis forced him to take repeated sick leaves and finally, in 1922, to retire. Kafka spent half his time after 1917 in sanatoriums and health resorts, his tuberculosis of the lungs finally spreading to the larynx.

His major works include: The Judgment (1912) [1], a long short story, which Kafka himself considered his decisive breakthrough; and The Metamorphosis (1915) [6], dealing again with the outsider, a son who suffers the literal and symbolic transformation into a huge, repulsive, fatally wounded insect. The Trial (1925) deals with a man persecuted and put to death by the inscrutable agencies of an unfathomable court of law. The Castle (1926) describes the relentless but futile efforts of the protagonist to gain recognition from the mysterious authorities ruling the village where he wants to establish himself. In all of these works, the lucid, concise style forms a striking contrast to the labyrinthine complexities, the anxietyladen absurdities, and the powerfully oppressive symbols of torment that are the substance of the writer's vision.

Kafka's Father, Hermann Kafka (1852-1931), son of the butcher Jacob Kafka and his wife, Franziska, was born and raised in Wossek, southern Bohemia. His family was poor and at the age of 18 he moved to Prague in hopes of bettering his situation. He succeeded, opening his own store and winning Julie Lowy, born March 23, 1856 in Podebray, the second child of Jakob Lowy, a well-to-do cloth merchant and brewer. Franz's relationship with his father was, to put it lightly, tempestuous, and would end up becoming the basis of much of his works. His education methods [refer to section 4.4. Remarks], as Franz had later reflected on, had almost ruined his son's life:

The Diaries
1911, Sunday, 19 July, slept, awoke, slept, awoke, miserable life.
I would not be so, if my education had penetrated into me as deeply as it wanted to. Perhaps my youth was too short for that, in which case, now in my forties, I still rejoice over its shortness with all my heart. That alone made it possible for me to have enough strength left to become conscious of the deprivations of my youth; further, to suffer through these deprivations; further, to reproach the past in all respects; and, finally, to have left a remnant of strength for myself. But all these strengths are, again, only a remnant of those that I possessed as a child, which exposed me more than others to the corrupters of youth, yes, a good racing chariot is the first to be pursued and overtaken by dust and wind, and its wheels fly over obstacles so that one might almost believe in love.
I often think it over and give my thoughts free rein without interfering, but I always come to the same conclusion: that my education has spoiled me more than all the people I know and more than I can conceive.

In his autobiographical work "Brief an der Vater" ("Letter to the Father") [5], written in 1919, Kafka blamed his father for his inability to break his family ties and establish an independent married life for himself. Kafka's father was the very opposite of Kafka himself: he was a down-to-earth shopkeeper who was obsessed with money and social success. In Kafka's imagination, this man belonged to a race of "giants": at the same time he hated and admired him. Yet, despite the obvious need to get away from his father, Kafka spent a major part of his life living with this awful man.

3. Preliminary Modelling

3.1. Scale; The Boundary Conditions

In this section, we are going to plot a graph which is to describe the relation between the attitude towards each other and time, by grading the incidents throughout his life. We shall consider the period from 1893 to 1923 [refer to section 4.1 Time Basis]. We would use the following scale throughout this paper: the feeling of one person to the other is always between two conditions-extreme hatred and extreme love. For this reason, we would quantify love (or hatred) to be between $+10$ and -10.

The boundary conditions will be set here, and it is a piece of information for both sections 3 and 4.

Definition 1. *t is the time parameter (unit is year, and $t = 0$ corresponds to the time when Kafka was 10).*

Definition 2. *$A_k(t)$ is the attitude or feeling of Franz towards his father (positive means respect, negative implies hatred).*

Definition 3. *$A_f(t)$ is the feeling of his father towards him.*

Definition 4. *$L(t)$ is a special function describing the education by Franz's father to him.*

Note: the necessity of $L(t)$ is discussed in section 4.4.

There is a lack of information on the time of $t = 0$. Hence we can only make up reasonable and convenient values for them.

It is believed that Franz didn't have much important events or big quarrels with his father before 10, or otherwise they will be described in some of his works. Therefore $L(0) = 0$, and both $A_k(0)$ and $A_f(0)$ would be close to zero. The following values are set much for the sake of convenient and accuracy, because we found that the curves obtained by setting these values agree quite well.

$$A_k(0) = 0.5$$
$$A_f(0) = 1$$
$$L(0) = 0$$

In this section, a polynomial equation $\delta(t)$ will be introduced. It is a function obtained by interpolation, through examining Kafka's various books at different times.

Our objective is to find a function A_k in sections 4 and 5, to fit $\delta(t)$ in section 4.

3.2. At the age of 15

In 1898, at the age of 15, Franz and his father had a fierce discussion about marriage [5]. This incident was crucial to the deterioration of their relationship.

"I am afraid that because in this sphere[marriage] everything I try is a failure, I shall also fail to make these attempts to marry comprehensible to you."
"..., all the negative forces that I have described as being the result in part of your method of upbringing, that is to say, the weakness, the lack of selfconfidence, the sense of guilt, and they positively drew a cordon between myself and marriage."

The incident, as Franz recalled, took place when the family went for a walk in Prague.

"I had begun talking about all this [marriage] mainly because it gave me pleasure at least to talk about it, and also out of curiosity, and finally to avenge myself somehow on the two of you for something or other. In keeping with your nature you took it quite simply, only saying something to the effect that you could give me advice about how I could go in for these things without danger. Perhaps I did want to lure just such an answer out of you; it was in keeping with the prurience of a child overfed with meat and all good things, physically inactive, everlastingly occupied with himself; but still, my outward sense of shame was so hurt by this for I believed it ought to be so hurt that against my will I could not go on talking to you about it and, with arrogant impudence, cut the conversation short."

"Cutting the conversation short" appears also in a later work, $\langle$ The Metamorphosis $\rangle$ [6], the protagonist Geogor fails to communicate with his family since he has turned into a gigantic insect. This indicates how great the impact of his father's opposition is, and we can realize how helpless and disappointing Franz felt at that time.

Then, the imposing image of his father appeared again. From the problem of marriage, which had always been the widest gap between them, we are again able to identify how he felt for his father-fear, while he considered himself a sinful man.

"The important thing was rather that you yourself remained outside your own advice, a married man, a pure man, above such things; this was probably intensified for me at the time by the fact that even marriage seemed to me shameless; and hence it was impossible for me to apply to my parents the general information I had picked up about marriage. Thus you became still purer, rose still higher. The thought that you might have given yourself similar advice before your marriage was to me utterly unthinkable. So there was hardly any smudge of earthly filth on you at all. And it was you who pushed me down into this filth just as though I were predestined to it with a few frank words. And so, if the world consisted only of me and you (a notion I was much inclined to have), then this purity of the world came to an end with you and, by virtue of your advice, the fifth began with me. In itself it was, of course, incomprehensible that you should thus condemn me; only old guilt, and profoundest contempt on your side, could explain it to me. And so again I was seized in my innermost being and very hard indeed. [5]"

Therefore an image conjures up in our mind, with a sinner crouching down on his knees, and standing in front is the only man of power. So it is possible that at the age of 15 this incident had pulled Franz further apart from his father, and the time when his feelings changed from love to hatred.

Mathematically, we can say $\delta(5) = 0$.

3.3. At the age of 18

In 1901, Franz entered the university. He studied Law, a choice actually made by his father. Nevertheless, when he graduated from university in 1906, he was given the freedom to choose his job, but soon we see this is only a choice of "no choice":

"For instance, the choice of a career. True, here you gave me complete freedom, in your magnanimous and, in this regard, even indulgent manner. Although here again you were conforming to the general method of treating sons in the Jewish middle class, which was the standard for you, or at least to the values of that class."

The following is about how his father's expectations stifled Franz's freedom.

"Finally, one of your misunderstandings concerning my person played a part in this too. In fact, out of paternal pride, ignorance of my real life, and conclusions drawn from my feebleness, you have always regarded me as particularly diligent."
"This was the state in which I was given the freedom of choice of a career. But was I still capable of making any use of such freedom? Had I still any confidence in my own capacity to achieve a real career?"

It is again his father's domineering character and misunderstanding that upset him.

Therefore, we see that Franz Kafka hated his father at that time, or $A_k(8) < 0$.

Also, his hatred was at the early developing stage, so we would assume that would not be too negative, approximately -2 or -3.

Approximately, let $\delta(8) \simeq -2.5$.

3.4. At the age of 29

In 1912, a crucial year when Kafka met and fell in love with Felice Bauer, and a prosperous time of literary development with his major works ⟨The Metamorphosis⟩ [6] and ⟨The Judgment⟩ [1] written. We believe that it is the time when their relationship is the worst.

Though Felice was already married, Franz still thought about marrying her shortly after their meeting. It can be seen from their frequent letters which were later collected in "Letters to Felice", and in ⟨The Judgment⟩ [1]. In the story, the protagonist Georg Bendemann, is engaged to Fraulein Frieda Brandenfeld. However, his father's objection and sentence him to death makes Georg committed suicide. This short story, which was written in only one night, truly reflects his feelings and the situation he faced at that time.

"Perhaps during his mother's lifetime his father's insistence on having everything his own way in the business had hindered him from developing any

real activity of his own."

"Hindrance", an obvious implication, and "of his own", give us a feeling that Franz at that time was much eager to get rid of his father's influence. Here is a description of how Georg met his father.

"At last he put the letter across a small lobby into his father's room, which he had not entered for months. There was in fact no need for him to enter it, since he saw his father daily at business and they took their midday meal together at an eating house; in the evening, it was true, each did as he pleased, yet even then, unless Georg-as mostly happened-went out with friends or, more recently, visited his fiancee, they always sat for a while, each with his newspaper, in their common sitting room. It surprised Georg how dark his father's room was even on this sunny morning.
'Ah, Georg,' said his father, rising at once to meet him. His heavy dressing gown swing open as he walked and the skirts of it fluttered around him-'My father is still a giant of a man,' said Georg to himself."

We can see that Georg and his father merely know each other. Also, his father is "a giant of a man", a symbol of absolute power. The intense relationship can further be seen below.

'I really only wanted to tell you,' went on Georg, who had been vacantly following the old man's movements, 'that I am now sending the news of my engagement to St. Petersburg.' He drew the letter a little way from his pocket and let it drop back again.
'To St. Petersburg?' asked his father.
'To my friend there,' said Georg, trying to meet his father's eye.-In business hours he's quite different, he was thinking, how solidly he sits here with his arms crossed.
'Oh ye. To your friend,' said his father, with peculiar emphasis.

It is known that his father hated Kafka being with his friends, as revealed in ⟨Letter to Father⟩ [5],

". . ., without any consideration for my feelings or respect for my judgment, to move in with abuse, defamation, and denigration. Innocent, childlike people, for instance, the Yiddish actor Lowy, had to pay for that. Without knowing him you compared him, in some dreadful way that I have now forgotten, to vermin and, as was so often the case with people I was fond of, you were automatically ready with the proverb of the dog and its fleas. Here

*I particularly recall the actor because at that time I made a note of your pro-
nouncements about him, with the comment: ' "This is how my father speaks
of my friend (whom he does not even know), simply because he is my friend.
I shall always be able to bring this up against him whenever he reproaches
me with the lack of a child's affection and gratitude." '*

In fact, from 1909 to 1912, Franz spent much time travelling with his friends
like Max Brod, to places such as Brescia, Riva, Berlin and Paris. Criticism
by his father possibly afflicted Franz at that time.

In this fierce encounter Georg turns out to be the loser who has no chance
and is unable to revolt against his father, and has to receive his sentence to
death with absolute obedience. It is worth paying careful attention to the
last sentence of this story, when Georg is about to kill himself:

"Dear parents, I have always loved you, all the same," and let himself drop.

Nothing is ever told so explicitly in all Kafka's works, this desperate vow
seems to be his innermost feelings. This shows that his feeling to his father
has not gone to absolute hatred or detestation. Therefore, we can deduce
that in 1912 his hatred for his father was about one less than the greatest
value:

$$\delta(19) = -9$$

3.5. At the age of 31

His first engagement to Felice Bauer in 1914 was opposed by his father. In
⟨Letter to Father⟩ [5], Franz mentioned how his father teased him after an-
nouncing his marriage plans.

*"Although both girls were chosen by chance, they were extraordinarily well
chosen. Again a sign of your complete misunderstanding, that you can be-
lieve that I timid, hesitant, suspicious can decide to marry in a flash, out of
delight over a blouse. Both marriages would rather have been commonsense
marriages, in so far as that means that day and night, the first time for
years, the second time for months, all my power of thought was concentrated
on the plan. Neither of the girls disappointed me, only I disappointed both
of them. My opinion of them is today exactly the same as when I wanted to
marry them."*
*"I will try to explain it in more detail. Here, in the attempt to marry,
two seemingly antagonistic elements in my relations with you unite more*

intensely than anywhere else. Marriage certainly is the pledge of the most acute form of self-liberation and independence. I would have a family, in my opinion the highest one can achieve, and so too the highest you have achieved; I would be your equal; all old and even new shame and tyranny would be mere history. It would be like a fairy tale, but precisely there lies the questionable element. It is too much; so much cannot be achieved. It is as if a person were a prisoner, and he had not only the intention to escape, which would perhaps be attainable, but also, and indeed simultaneously, the intention to rebuild the prison as a pleasure dome for himself. But if he escapes, he cannot rebuild, and if he rebuilds, he cannot escape. If I, in the particular unhappy relationship in which I stand to you, want to become independent, I must do something that will have, if possible, no connection with you at all; though marrying is the greatest thing of all and provides the most honorable independence, it also stands at the same time in the closest relation to you. To try to get out of this quandary has therefore a touch of madness about it, and every attempt is punished by being driven almost mad."

It seems that, to a large extent, Franz would still cancel his engagements even his father didn't oppose to it. "This very comparison proves that I certainly do not mean to say that you drove me away from marriage by your example, as you had driven me away from your business. [5]" From this evidence we can see that their relationship is still tense. That is, $A_k(t)$ approaches zero at a very slow speed at $t \simeq 20$. This, together with the preceding on the analysis on $A_k(19)$, convinces us that $\delta'(19) = 0$.

3.6. At the age of 34

One of his short story, ⟨A Crossbreed⟩ (1917) [2], gives us clues that the feeling of alienation had only been merely relieved.

"I have a curious animal, half kitten, half lamb. It is an heirloom that belonged to my father."

The "half kitten, half lamb" animal is Franz himself, which is "an heirloom" belonged to his father. The crossbreed has two characteristics, incompatible to each other. This symbolizes Lowy and Kafka characteristics in him, and also the conflicting feeling for his father - love and hatred.

"Sometimes the children bring cats with them; once they actually brought two lambs. But against all their hopes there was no sense of recognition.

The animals gazed calmly at each other with their animal eyes, and obviously accepted their reciprocal existence as a divine fact."

It is quite astonishing that Franz mentioned he [2] had been independent of his father, a mental attempt to expunge his father's influence. But at the end of the story, this attempt finally failed:

"Perhaps the knife of the butcher would be a release for this animal; but as it is a legacy I must deny it that." What is the legacy, then? "I did not inherit much from my father, but this legacy is quite remarkable."

An animal with two characteristics can never become one of its parts. It is in nature "worthless", so "the knife of the butcher" can put an end to its doomed life. Here Franz made a sarcastic joke on his father, as he said "this legacy is quite remarkable". Its fate and tragedy are sealed and unalterable, for it is inherited from its father. It is the father who creates the whole misfortune, and it is he who to be blamed. Yet, we can also see that Franz had tried his best to forgive his father, we will approximate $A_k(24)$ to be $\delta(19) - 0.5$, i.e. $\delta(24) = -8.5$.

3.7. At the age of 36

In 1919, the ⟨Letter to Father⟩ [5] was written, and he was engaged to Julie Wohryzek. Again the engagement was objected by his father.

"You said to me something like this: ' "She probably put on a fancy blouse, something these Prague Jewesses are good at, and right away, of course, you decided to marry her. And that as fast as possible, in a week, tomorrow, today. I can't understand you: after all, you're a grown man, you live in the city, and you don't know what to do but marry the first girl who comes along. Isn't there anything else you can do? If you're frightened, I'll go with you." [5] '

It is very possible that his father was hinting at finding a prostitute. Apparently Franz could not stand such mockings, and was furious with his father. However, comparing with the last few proposed engagements, Franz responded to his father's objection much more calmly.

"This guiltlessness on both sides I can judge especially well because a similar clash between us occurred some twenty years later, in quite different circumstances horrible in itself but much less damaging or what was there in me,

the thirty-sixyear- old, that could still be damaged?"

This letter can be seen a desire of Franz to make reconciliations with his father. It means that Franz at that time, unlike what he behaved before, wanted to communicate with him, which could only be possible if his bad feelings of his father were relieved. Comparing with the period of $t = 19-24$, the drop is expected to be greater (1). i.e. $\delta(26) = -7.5$

3.8. At the age of 40

Franz was writing $\langle$The Burrow$\rangle$ [TB] at that time. He had a time with Dora Dymant, an Orthodox Jewish girl from Poland aged 19, in Berlin, which can be considered the beginning of his physical separation from his father. This physical separation, again, as one may expect, appears in his work. The narrator in $\langle$The Burrow$\rangle$, identity unknown to everyone, builds a burrow for himself. It is exactly what Franz was doing at that moment. The burrow is on one hand seen as the haven of this tiny, vulnerable creature, while on the other it is itself not resistant to any invasion.

"But the most beautiful thing about my burrow is the stillness. Of course, that is deceptive. At any moment it may be shattered and then all will be over. For the time being, however, the silence is still with me."

He appreciates his work, though he believes that repairs and regular checking are necessary to rule out any possibilities of an invasion. He is vigilant all the time. However to him, there are too many enemies that he has to face, and finally he is exhausted from laborious work.

"And it is not only by external enemies that I am threatened. There are also enemies in the bowels of the inner earth. I have never seen them, but legend tells of them and I firmly believe in them. They are creatures of the inner earth; no even legend can describe them. Their very victims can scarcely have seen them; they come, you hear the scratching of their claws just under you in the ground, which is their element, and already you are lost. Here it is of no avail to console yourself with the thought that you are in your own house; far rather are you in theirs."

We can notice that at least one of his enemies [Franz's] is his father. The burrow is built partly because of him, to break off from his father's dominant position in his life.

"..., and even now, at the zenith of my life, I can scarcely pass an hour in complete tranquility."

He is still very sensitive to any disturbances or invasions which would upset his life, and also aware of his own movements which would alarm his father. Nevertheless, at least, Franz managed to start a new life with Dora, a real burrow in Berlin. The attempt had been made, and he seemed to be satisfied with the situation. At the end of the story, unlike the others, all efforts made somehow manage to save the creature's life.

"The more I reflect upon it the more improbable does it seem to me that the beast has even heard me; it is possible, although in some other way, but it has certainly never heard me. So long as I still knew nothing about it, it simply cannot have heard me, for at that time I kept very quiet, nothing could be more quiet than my return to the burrow; afterwards, when I dug the experimental trenches, perhaps it could have heard me, though my style of digging makes very little noise; but if it had heard me I must have noticed some sign of it, the beast must at least have stopped its work every now and then to listen. But all remained unchanged."

Finally nothing happens. It is the first time in his major works that the vulnerable protagonist is able to resist the inexorable power with his almost futile struggle.

This can easily be seen to be a description of Franz himself. After 40 years of torture by his father, he finally can get out of that and "dig his own burrow". He could lead his own life since then. Therefore his feelings, or hatred towards his father should have another great drop (1.5). i.e. $\delta(30) = -6$.

3.9. The Graph

To sum up, we have graded the values of, Franz's feelings, at a number of
ages: 10, 15, 18, 29, 34, 36, and 40:

$$\delta(0) = 0.5$$
$$\delta(5) = 0$$
$$\delta(8) = -2.5$$
$$\delta(19) = -9$$
$$\delta(24) = -8.5$$
$$\delta(26) = -7.5$$
$$\delta(30) = -6.$$

And the value of its derivative is only known at one time-age= 29:

$$\delta'(19) = 0$$
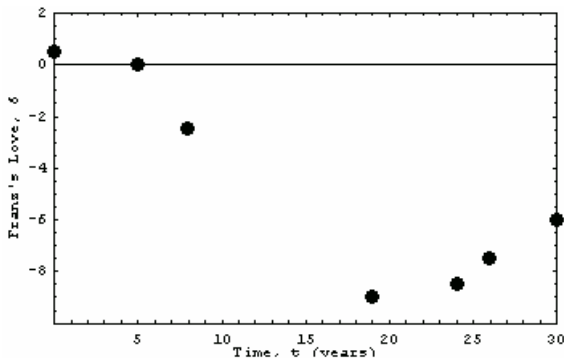
The seven points are plotted in the following curve:

FIGURE 3A

Now it is necessary to find out a curve that satisfied all of the above con-
ditions. As there are eight conditions, we let $\delta(x)$, to be a polynomial with
a degree of seven. Therefore $\delta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 +
a_6 t^6 + a_7 t^7$, where a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7 are all constants.

Substituting all the values back, we get

$$\delta(0) = 0.5 \quad \Rightarrow a_0 = 0.5,$$

$$\delta(5) = 0 \quad \Rightarrow a_0 + 5a_1 + 25a_2 + 125a_3 + 625a_4 + 3125a_5$$
$$+ 15625a_6 + 78125a_7 = 0,$$

$$\delta(8) = -2.5 \quad \Rightarrow a_0 + 8a_1 + 64a_2 + 512a_3 + 4096a_4 + 32768a_5$$
$$+ 262144a_6 + 2097152a_7 = -2.5,$$

$$\delta(19) = -9 \quad \Rightarrow a_0 + 19a_1 + 361a_2 + 6859a_3 + 130321a_4 + 2476099a_5$$
$$+ 47045881a_6 + 893871739a_7 = -9,$$

$$\delta(24) = -8.5 \quad \Rightarrow a_0 + 24a_1 + 576a_2 + 13824a_3 + 331776a_4 + 7962624a_5$$
$$+ 191102976a_6 + 4586471424a_7 = -8.5,$$

$$\delta(26) = -7.5 \quad \Rightarrow a_0 + 26a_1 + 676a_2 + 17576a_3 + 456976a_4 + 11881376a_5$$
$$+ 308915776a_6 + 8031810176a_7 = -7.5,$$

$$\delta(30) = -6 \quad \Rightarrow a_0 + 30a_1 + 900a_2 + 27000a_3 + 810000a_4 + 24300000a_5$$
$$+ 729000000a_6 + 21870000000a_7 = -6,$$

$$\delta'(19) = 0 \quad \Rightarrow a_1 + 38a_2 + 1083a_3 + 27436a_4 + 651605a_5$$
$$+ 1485659a_6 + 329321167a_7 = 0.$$

On solving, we get

$$a_0 = 0.5,$$
$$a_1 = -1.42154,$$
$$a_2 = 0.854092,$$
$$a_3 = -0.188084,$$
$$a_4 = 0.0177389,$$
$$a_5 = -0.000835707,$$
$$a_6 = 0.000019503,$$
$$a_7 = -1.79825 \times 10^{-7}.$$

Therefore,

$$\delta(t) = 0.5 - 1.42154t + 0.854092t^2 - 0.188084t^3 + 0.0177389t^4$$
$$0.00083507t^5 + 0.000019503t^6 - 1.79825 \times 10^{-7}t^7.$$
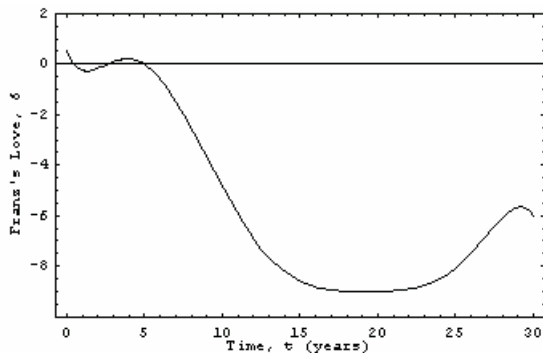
Then we plot the curve,

FIGURE 3B

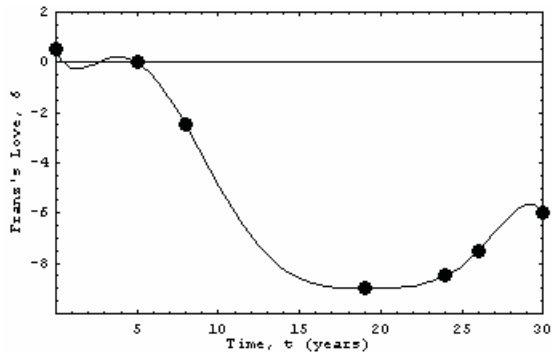The following graph is the combination of Figure 3A and Figure 3B:

FIGURE 3C

4. Determination of the coefficients & functions

4.1. Time Basis

Franz Kafka was born in 1883, and died in 1924. He spend most of his life living with his father in Prague (with occasional excursions into mountains, trips, and staying in sanatoriums) until 1923, when he met Dora Dymant. In September Franz moved out of his parents' apartment and went to Berlin

with Dora, despite a few attempts from 1915-1917 to have his own place.

In the following, we will only study the time starting when he was 10, and ending when he was 40. For example, $t = 15$ corresponds to 1908, when Franz was 25 years old. The period from age 40 onwards is not included since there is no actual evidence showing any communication between them at that time. Besides, as Franz was too young from 0 to 10, and his mind was not mature enough to be of great interest, the period is also not included.

4.2. Model

We would use Definitions 1-4 in Section 3.1., together with the following definitions:

Definition 5. $-\tau_1$, $-\tau_2$ and $-\tau_3$ are the forgetting coefficients.

Definition 6. D stands for the intrinsic difference between the character of Franz and his father: one is a great writer, one is a rude man.

Definition 7. $R_k()$ and $R_f()$ are reaction functions describing their response to the other's attitude towards them.

Definition 8. $M_k()$ and $M_f()$ are the humanity functions that explain the human nature to forgive and love their parents or children.

Definition 9. β is Franz's admiration for his father, because of his father's success in his business.

Then their relationships can be modelled by the following system of equations:

$$\frac{dA_k}{dt} = -\tau_1 A_k - \alpha_1 D + R_k(A_f) + M_k(A_k) + \frac{\beta}{1-L},$$

$$\frac{dA_f}{dt} = -\tau_2 A_f - \alpha_2 D + R_f(A_k) + M_f(A_f),$$

$$\frac{dL}{dt} = -\tau_3 L + N(A_f).$$

Explanations:

1. $-\tau_1 A_k$, $-\tau_2 A_f$ and $-\tau_3 L$ characterizes the *forgetting process* of each one, as humans tend to forget their love or hatred towards the others.
2. D caused (directly or indirectly) each one tend to hate the other ($\alpha_1 D$ and $\alpha_2 D$ respectively).
3. β can be attenuated by his father's intense scolding, irony and threats on him, characterized by $L(t)$.

4.3. Assumptions

1. Omit Franz Kafka's sickness.
2. Omit the aging of his father.
3. Omit many external factors, for instance, belief in Judaism, wars at that time, etc...

4.4. Remarks

Franz's father did not read any of his books:

'My vanity, my ambition did suffer under your soon proverbial way of hailing the arrival of my books: "Put it on my bedside table!" ' [5]

Therefore, his father would never realize his dominating influence from Franz's works, and he would not be influenced by his works at all.

Our special function, $L(t)$, is needed because his father's method of education was very bad, and Franz had always said that this had made him "permanently impotent". We believe that this is worth to be studied to a deeper extent, thus we open another function for it.

5. Finalized model and graphing

5.1. The *forgetting coefficients*

On the first sight, we know $\tau_2 > \tau_1$, because Franz's father was a successful and busy businessman, which could in some way weaken his feelings towards his son as he had no time taking care of Franz.

Set $\tau_2 = 1$. However, it is highly plausible that τ_1 will vary, because as a child grows up, his mind becomes more mature and the love or hatred in his mind can fade away faster.

And we don't want it to get greater that τ_2, so set $-\tau_1 = 0.2\sqrt[3]{t}$, such that τ_1 first increases faster in the beginning, and then slows down as t is large.

Now for τ_3. Franz, as a writer, was much aware of the careful use of language. He felt that language had been insulted and offended by his father's foul words. So we take it to be half of τ_1, i.e. $\tau_3 = 0.5$.

5.2. The terms $-\alpha_1 D$ and $-\alpha_2 D$

Although they have very different individualities, their response to this is quite different. The father, on one hand, was angry about the son's introversion and unwillingness to find a job. Franz Kafka, on the other hand, was furious about the father's rudeness and intervention on his choice of studies in university, on finding jobs, and on his marriage. In the letter [5], Franz said:

"Compare the two of us: I, to put it in a very much abbreviated form, a Lowy with a certain Kafka component, which, however, is not set in motion by the Kafka will to life, business, and conquest, but by a Lowyish spur that impels more secretly, more diffidently, and in another direction, and which often fails to work entirely. You, on the other hand, a true Kafka in strength, health, appetite, loudness of voice, eloquence, self-satisfaction, worldly dominance, endurance, presence of mind, knowledge of human nature, a certain way of doing things on a grand scale, of course also with all the defects and weaknesses that go with these advantages and into which your temperament and sometimes your hot temper drive you."

Lowy is the surname of her mother, and to Franz it represents an artistic sense. From the above passage, we can assume that $-\alpha_1 D$ is 4 times $-\alpha_2 D$. Also, their difference in the individualities should play a major role in their development of relationship, so we set higher values.

Let $-\alpha_2 D = -0.5$, so $-\alpha_1 D = -2$.

5.3. The *reaction functions* $R_k()$ and $R_f()$

As people love to be loved and hate to be hated, we can use linear functions to approximate $R_k()$ and $R_f()$. But it is worth noting that as Franz was very introverted and so his father should be less sensitive about his feelings. It is, again, verified in "Letter to Father" [5]:

"The impossibility of getting on calmly together had one more result, actually a very natural one: I lost the capacity to talk"

Again let R_k be the identity function, and R_f be half of that:

$$R_k : x \to x$$
$$R_f : x \to 0.5x$$

5.4. The *humanity functions* $M_k()$ and $M_f()$

Because they were father and son, they two will tend to forgive the other. In addition, with reference to Sigmund Freud's psychoanalysis, the effect of Oedipus Complex (See Appendix 10.1.) is also included. That is, the son [Franz] would be jealous of the father as he possessed the mother, so Franz's love for his father would be reduced. It is expected that the two functions should have the following properties:

1. $M_k(a) = M_f(a) = 0$ if $a > 0$.
2. $M_k(b) > M_k(a)$ if $b < a < 0$. This also holds for $M_f(a)$.
3. $M_k(a) \ll M_f(a)$ because of the effect of Oedipus Complex and he was insulted again and again by his father.
4. $M_k(a)$ is larger if t is larger. This is because as a child grows up, he would love his father more. Hence we will change $M_k()$ to be a multi-variable function.

Here we would introduce the UnitStep Function [refer to Appendix 10.2.]

Definition 1. *t is the time parameter (unit is year, and $t = 0$ corresponds to the time when Kafka was 10).*

Definition 10.
$$\text{UnitStep}(x) = \begin{cases} 0, & \text{if } x < 0, \\ 1, & \text{if } x \geqslant 0. \end{cases}$$

By the properties above and Definition 10, one can define:
$$M_k : (x, t) \to 0.001t(1 - \text{UnitStep}(x))(-x)$$
$$M_f : x \to 0.5(1 - \text{UnitStep}(x))(-x)$$

5.5. The *term of admiration*, $\dfrac{\beta}{1 - L}$

To assign a value to β, we will assume that it is as important as the term characterizing their different individualities, because Franz Kafka himself had said that his respect to his father was because of his father's success in

business, sinewy build and determined mind.

"In keeping, furthermore, was your intellectual domination. You had worked your way so far up by your own energies alone, and as a result you had unbounded confidence in your opinion. That was not yet so dazzling for me, a child as later for the boy growing up. From your armchair you ruled the world. [5]*"*

Set $\beta = 2$.

5.6. Franz's father's education by insults $N()$

As described by Franz in his letter [5], his father used an education method of "abuse, threats, irony, spiteful laughter" and, foul language.

"I'm not going to say, of course, that I have become what I am only as a result of your influence. That would be very much exaggerated (and I am indeed inclined to this exaggeration). It is indeed quite possible that even if I had grown up entirely free from your influence I still could not have become a person after your own heart. I should probably have still become a weakly, timid, hesitant, restless person, neither Robert Kafka nor Karl Hermann, but yet quite different from what I really am, and we might have got on with each other excellently."

Also, in 1911, in his Diaries [8], Franz told of the impact of his father's education.

"When I think about it, I must say that my education has done me great harm in some respects."
"For me as a child everything you called out to me was positively a heavenly commandment, I never forgot it, it remained for me the most important means of forming a judgment of the world, above all of forming a judgment of you yourself, and there you failed entirely."

We will assume that when A_f is positive, his father will not affect L, but when A_f becomes more and more negative, his education to Kafka will be more and more poor, using even more "abuse, threats, irony, spiteful laughter". That, we assume, is proportional to A_f.

Set $N : x \to (1 - \text{UnitStep}(x))(x)$ with Definition 10.

6. Analysis

6.1. The Final Model

$$\frac{dA_k}{dt} = -0.2\sqrt[3]{t}A_k - 2 + A_f + 0.001t(1 - \text{UnitStep}(A_k))(-A_k) + \frac{2}{1 - L}$$

$$\frac{dA_f}{dt} = -A_f - 0.5 + 0.5A_k + 0.5(1 - \text{UnitStep}(A_f))(-A_f)$$

$$\frac{dL}{dt} = -0.5L + (1 - \text{UnitStep}(A_f))(A_f)$$

with boundary conditions:

$$A_k(0) = 0.5 \quad [\text{refer to 3.1. Scale; The Boundary Conditions}]$$
$$A_f(0) = 1$$
$$L(0) = 0$$

6.2. Graphing

Using computer to approximate solutions, [refer to Appendix 10.3.1 for the *Mathematica* Code]
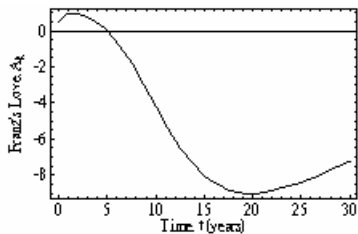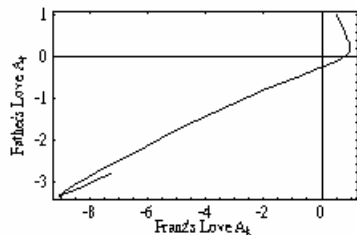

FIGURE 6A
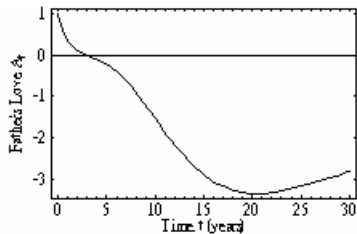
FIGURE 6B
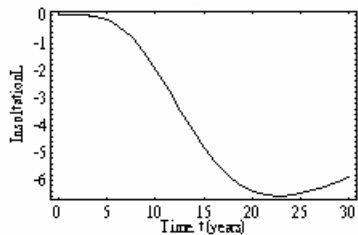
FIGURE 6C

FIGURE 6D

7. Analysis

7.1. Comparison

The following are the comparisons between the results of Section 3 and Section 5

Figure 7A: Figure 3B (Blue) + Figure 6A (orange)
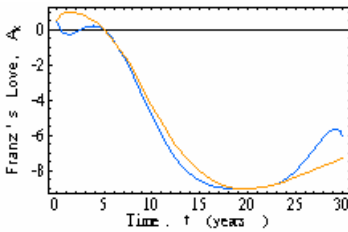
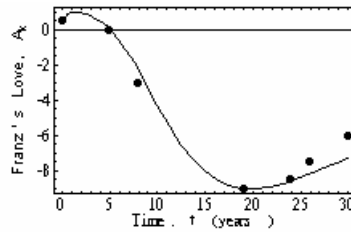Figure 7B: Figure 3A + Figure 6A

FIGURE 7A

FIGURE 7B

7.2. Analysis

From the two graphs, we see that our model is roughly good. Then we see that the deviations of our model from $\delta(t)$ are less than 1 for most of the time. This confirms that our model is able to describe the relationship quite well.

However, although our model almost fits the data before $t = 20$, we can still see that there is a trend of increasing error starting from then. We believe that it is due to increasing age of Franz. At different age people have different perceptions in life, and an older person is considered to be more sophisticated. Therefore it is of higher possibility that error occur. Also, we encounter the mature stage of his literary works, which are much more difficult to comprehend and decipher. This would lead to errors if we misinterpret the works.

8. Extension

First part of this section is to test the resultant graph if some parameters are altered, thus find out the possible ways to improve the relationship.

Then we will try to extend our model to a more general condition: the relationship between two persons.

8.1. Communication

From our intuition, communication between parents and children is important. How is this reflected in the equations?

If they communicate more, his father can know more about Franz's feelings. Hence the *reaction function* should be larger. If we set the coefficient to be slightly larger, say 0.8:

$$\frac{dA_k}{dt} = -0.2\sqrt[3]{t}A_k - 2 + A_f + 0.001t(1 - \text{UnitStep}(A_k))(-A_k) + \frac{2}{1-L}$$

$$\frac{dA_f}{dt} = -A_f - 0.5 + 0.8A_k + 0.5(1 - \text{UnitStep}(A_f))(-A_f)$$

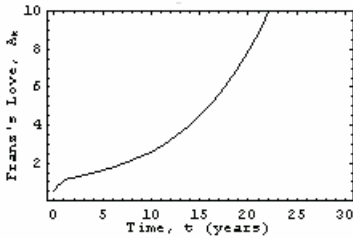$$\frac{dL}{dt} = -0.5L + (1 - \text{UnitStep}(A_f))(A_f)$$
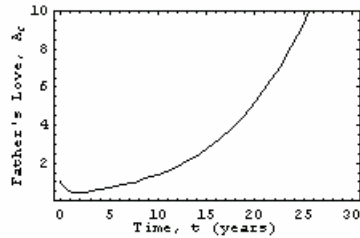

FIGURE 8A

FIGURE 8B

[refer to Appendix 10.3.2 for the *Mathematica* Code] which shows that they would tend to have good relationship.

Therefore our assertion is justified.

8.2. Attitude of Franz's father

As described before, Franz's father was a barbarian, often insulted and forced Franz to do things that he was unwilling to do. What would happen if he treated his son better?

Firstly, the humanity function M_k is larger since Franz feels closer with his father. Then $N()$ will be smaller if his father scolds him less and uses better education methods.

Let's assume that the former one is magnified 10 times and the latter one is diminished by 10 times.

$$\frac{dA_k}{dt} = -0.2\sqrt[3]{t}A_k - 2 + A_f + 0.01t(1 - \text{UnitStep}(A_k))(-A_k) + \frac{2}{1 - L}$$
$$\frac{dA_f}{dt} = -A_f - 0.5 + 0.5A_k + 0.5(1 - \text{UnitStep}(A_f))(-A_f)$$
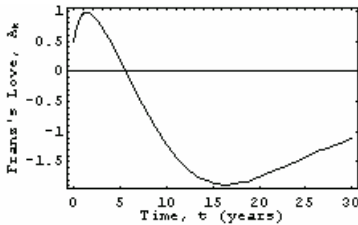$$\frac{dL}{dt} = -0.5L + 0.1(1 - \text{UnitStep}(A_f))(A_f)$$
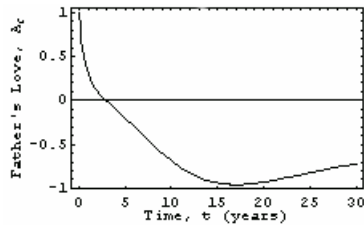

FIGURE 8C   FIGURE 8D

[refer to Appendix 10.3.3 for the *Mathematica* Code]

Hence the magnitudes of the two curves are smaller, meaning they hate each other less.

Therefore our assertion is justified.

8.3. Generalization

Rinaldi's paper [4] describes the relationship of Laura and Petrarch very well by ordinary differential equations with the following terms: *forgetting processes*, *terms of appeal*, and *reaction functions*, though the functions used

there were comparatively simpler. In the above, this methodology is extended to the case of Kafka, and the duration of analysis is 30 years-ten more than that of [4], and we have taken more terms into considerations. Upon the success of this model, we seek to find out the terms that should be taken into account in formulating the system of ordinary differential equations that describe the interpersonal relationship between two people. We suspect that the following are some, if not all, of the terms:

1. The *forgetting process*
   As considered in both of our paper and Rinaldi's [4], this charcterizes the cooling passion of each person, and is always negative.
2. The *reaction functions*
   This type of functions is crucial in describing how one would react to the other's love/hatred. Rinaldi argued that this can be either positive or negative.
3. The *individualities of each other*
   Sometimes people hate the other because of the other's characters. But in the case of lovers, each one's individualities attract the other. In this paper, this is reflected in the negative terms $-\alpha_1 D$ and $-\alpha_2 D$. However, in [4], this is reflected in the terms of appeal (positive for one and negative for the other).
4. The *humanity functions*
   This term, included only in this paper, is necessary only if the two people have close family relationships with each other. As part of human nature, people tends to forgive and love their relatives. This term is positive.
5. The *term ofadmiration/detestation*
   Again a term included only in this paper, this term shows the effect of one's achievements or criminals on the other. It is believed that Rinaldi has omitted this term, since Petrarch was a famous Italian poet [4].

9. Conclusion

In this article, we have analysed the complicated relationship between Kafka and his father by a system of ordinary differential equations. We have found that our approach has been fairly reliable in predicting their relationship. We have also proved that communication between father and son would be crucial in developing good relationships. Also, in this special case of Kafka, we have found that his father's attitude also affect their relationship.

Throughout his 41 years of life Franz spent most of the time with his beloved

and though terrible father, and for nearly 25 years they had had a negative relationship according to our results. This intriguing real life story, interwoven with a son of diffidence and a father of tyranny, is doomed to be a tragedy. Their conflicts were, and are, going on for ever. However, as we all know, all his works sprang out like a fountain which flows and influences the contemporaries and even the next generation, from the sole source of inspiration-his life with his father. It is ironic; it gives him fame and reputation after his death. Without his father, Franz Kafka could not become the Franz Kafka we know.

With our love in both literature and mathematics we are eager to materialize our wish to combine the two together. We have, again, proved that there is no contradiction between science and literature. We have successfully described the complex interpersonal relationship using elegant mathematical methods. Though their scope is totally different and not related, what the writers and mathematicians pursue in common is the truth.

We believe that our work is pioneering in describing interpersonal relationships by first-order ordinary differential equations. However, we believe that our mathematical techniques are not sophisticated enough and we may have omitted many important factors. We hope that other mathematicians and sociologists can continue to work in this field and to find out some more general formulae governing the society. Besides, we would be very grateful if our work helps inspire people with a caring and friendly attitude towards the others, which is the only key to the building of a harmonious society, and a better place to live in.

10. Appendix

10.1. The Effect of Oedipus Complex

The Oedipus complex is developed by Sigmund Freud, a concept in order to explain the maturation of the infant identification with the father and desire for the mother. It is based on the Greek myth of Oedipus who kills his father Laius and marries his mother Locaste.

The traditional paradigm in a (male) child's psychological coming-into-being is to first select the mother as the object of libidinal investment. This arouses the father's anger, and the infant surmises that the most probable form of this is castration. The infant internalizes the rules pronounced by his father. The father now becomes the figure of identification as the child wants

to have his phallus, but resigns from his attempts to take the mother, shifting his libidinal attention to new objects of desire.

French psychoanalyst Jacques Lacan claimed that the position of the father could never be held by the infant. On one hand the infant must identify with the father, in order to participate in sexual relations. However the infant could also never become the father as this would imply sexual relations with the mother. Through the dictates on the one hand to be the father and on the other not to, the father is elevated to an ideal. He is no longer a real material father, but a function of a father.

Therefore, in short, due to "castration anxiety" the son develops a fear of his father, which eventually worsens their relationship.

10.2. The UnitStep Function

Also known as the Heaviside function, it is commonly denoted by $U(x)$ or $H(x)$. It is most commonly used in defining periodic functions for Fourier Series Its graph is as follow:
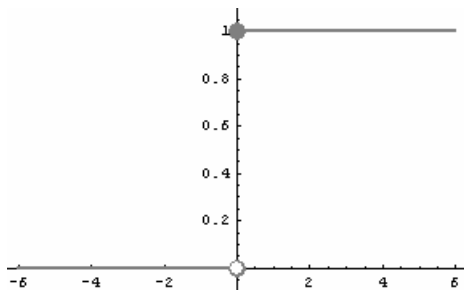

FIGURE 10A

Its derivative is a special DiracDelta Function, whose value is zero, except at $x = 0$, where it has a singularity.

Also, $\int_{\infty}^{\infty} \text{DiracDelta}(x)dx = 1$, as expected.

10.3. The Mathematica Code

10.3.1. Fig.6A, 6B, 6C, and 6D

```
NDSolve[{K'[t]==-0.2∛tK[t]-2+F[t]+.001t(1-UnitStep[K[t]])(-K[t])+  2
                                                               ─────,
                                                               1-L[t]
        F'[t]==-F[t]-.5+.5K[t]+.5(1-UnitStep[F[t]])(-F[t]),
        L'[t]==-0.5L[t]+(1-UnitStep[F[t]])(F[t]),
        K[0]==.5, F[0]==1, L[0]==0},
        {K[t],F[t],L[t]},
        {t,0,30}];
tmp1[t_],tmp2[t_],tmp3[t_]=%[[1,{1,2,3},2]];
K1=Plot[tmp1[t],{t,0,30},Frame→True,
    FrameLabel→{"Time, t (years)","Franz's Love, A_k"},PlotLabel→"Figure 6A",
    DisplayFunction→Identity];
K2=Plot[tmp2[t],{t,0,30},Frame→True,
    FrameLabel→{"Time, t (years)","Father's Love, A_f"},PlotLabel→"Figure 6B",
    DisplayFunction→Identity];
K3=Plot[tmp3[t],{t,0,30},Frame→True,
    FrameLabel→{"Time, t (years)","Insultation, L"},PlotLabel→"Figure 6C",
    DisplayFunction→Identity];
K4=ParametricPlot[{tmp1[t],tmp2[t]},{t,0,30},Frame→True,
    FrameLabel→{"Franz's Love, A_k","Father's Love, A_f"},PlotLabel→"Figure 6D",
    DisplayFunction→Identity];
Show[GraphicsArray[{{K1,K2},{K3,K4}}],DisplayFunction→$DisplayFunction];
```

10.3.2. Fig. 8A and 8B

```
NDSolve[{K'[t]==-0.2∛tK[t]-2+1F[t]+.001t(1-UnitStep[K[t]])(-K[t])+  2
                                                                 ─────,
                                                                 1-L[t]
        F'[t]==-F[t]-.5+0.8K[t]+.5(1-UnitStep[F[t]])(-F[t]),
        L'[t]==-0.5L[t]+(1-UnitStep[F[t]])(F[t]),
        K[0]==.5, F[0]==1, L[0]==0},
        {K[t],F[t],L[t]},
        {t,0,30}];
tmp1[t_],tmp2[t_],tmp3[t_]=%[[1,{1,2,3},2]];
AK1=Plot[tmp1[t],{t,0,30},Frame→True,
    FrameLabel→{"Time, t (years)","Franz's Love, A_k"},PlotLabel→"Figure 8A",
    PlotRange→{0,10},DisplayFunction→Identity];
AK2=Plot[tmp2[t],{t,0,30},Frame→True,
    FrameLabel→{"Time, t (years)","Father's Love, A_f"},PlotLabel→"Figure 8B",
    PlotRange→{0,10},DisplayFunction→Identity];
AK3=Plot[tmp3[t],{t,0,30},Frame→True,
    FrameLabel→{"Time, t (years)","Insultation, L"},
    DisplayFunction→Identity];
AK4=ParametricPlot[{tmp1[t],tmp2[t]},{t,0,30},Frame→True,
    FrameLabel→{"Franz Kafka to father","Father to Franz Kafka"},
    DisplayFunction→Identity];
Show[GraphicsArray[{{AK1,AK2}}],DisplayFunction→$DisplayFunction];
```

10.3.3. Fig. 8C and 8D

```
NDSolve[{K'[t]==-0.2∛t K[t]-2+1F[t]+.01t(1-UnitStep[K[t]])(-K[t])+ 2/(1-L[t]) ,
        F'[t]==-F[t]-.5+0.5K[t]+.5(1-UnitStep[F[t]])(-F[t]),
        L'[t]==-0.5L[t]+(1-UnitStep[F[t]])(F[t]),
        K[0]==.5, F[0]==1, L[0]==0},
        {K[t],F[t],L[t]},
        {t,0,30}];
tmp1[t_],tmp2[t_],tmp3[t_]=%[[1,{1,2,3},2]];
AK5=Plot[tmp1[t],{t,0,30},Frame→True,
    FrameLabel→{"Time, t (years)","Franz's Love, Aₖ"},PlotLabel→"Figure 8C",
    DisplayFunction→Identity];
AK6=Plot[tmp2[t],{t,0,30},Frame→True,
    FrameLabel→{"Time, t (years)","Father's Love, A_f"},PlotLabel→"Figure 8D",
    DisplayFunction→Identity];
AK7=Plot[tmp3[t],{t,0,30},Frame→True,
    FrameLabel→{"Time, t (years)","Insultation, L"},
    DisplayFunction→Identity];
AK8=ParametricPlot[{tmp1[t],tmp2[t]},{t,0,30},Frame→True,
    FrameLabel→{"Franz Kafka to father","Father to Franz Kafka"},
    DisplayFunction→Identity];
Show[GraphicsArray[{{AK5,AK6}}],DisplayFunction→$DisplayFunction];
```

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