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BEST POSITIONS FOR INSTALLING LIFTS IN A SHOPPING MALL

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Abstract. A shopping mall can be divided into different regions with different numbers of people. After a person has finished shopping in a shop the person may want to go to other levels but there are only limited numbers of lifts in certain positions. In our project, we attempted to find the best position for installing lifts in a shopping mall such that the total walking distances for people to reach the lifts can be minimized.

1. Introduction

Firstly, we consider the simplest case - a mall in which shops are arranged one dimensionally in a straight line and there is one lift.

Then, we look into the case that shops are arranged two dimensionally and there is one lift. Then, we further expand the idea to a shopping mall with shops arranged three dimensionally and there is only one lift.

After considering cases with one lift only, we consider cases in which there are more lifts. We started with the simplest one in which there are two lifts to be installed in a one dimensional shopping mall.

We have also written a PASCAL program for finding the best positions for installing two lifts in a one-dimensional shopping mall.

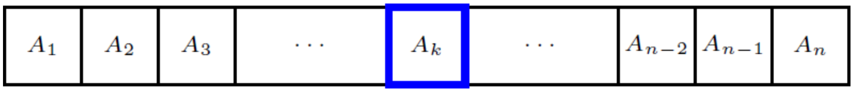
[1]This work is done under the supervision of the authors' teacher, Mr. Wing-Kwong Wong.

2. Best Positions for Lifts in Shopping Malls

Conditions

The model of shopping mall we use is a mall in the shape of a rectangular block in which all different levels are identical. On different levels, the mall can be divided into different square regions. All levels of the shopping mall are rectangular on a xy-plane; all people can only walk in straight lines along directions of x-axis and y-axis.

2.1. One dimensional shopping mall with one lift



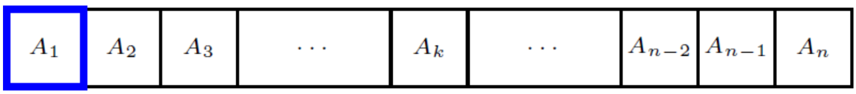
Consider a shopping mall divided into n square regions, let the length of sides of each region be 1.

A_i denotes the number of people in the region and blue square is the region in which the lift is located.

If there is a lift at region k, then

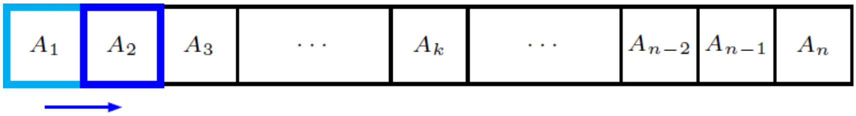
$$\text{the average distance between people and lifts} = \frac{\sum_{i=1}^{n} A_i \times |k-i|}{\sum_{i=1}^{n} A_i}.$$

However, it is not easy to find the best position from the formula above directly. Consider the extreme case: $k = 1$



If we move the lift to the right for 1 unit, then average distances between each people in regions 2 to n decreases by 1, but that between people in region 1 increases by 1. The total distance decreases by $(-A_1 + A_2 + A_3 +$

$\cdots + A_{n-1} + A_n)$.



Each time we move the lift rightward for 1 unit, the total distance decreases by

$$-\sum_{i=1}^{k} A_i + \sum_{i=k+1}^{n} A_i.$$

This implies that when $A_1 + \cdots + A_k = A_{k+1} + \cdots + A_n$, further shifting does not decrease the distance, but the opposite. We can therefore conclude that the best position is one that divides the number of people into two equal halves with one half on each side.

2.2. Two dimensional shopping mall with one lift

First put the shopping mall on a xy-plane. Let (x_0, y_0) be the location of the lift, then a people at the position (x_1, y_1) has to walk a distance of $|x_0 - x_1| + |y_0 - y_1|$ to reach the lift. Note that $|x_0 - x_1|$ is independent of the y-coordinate of the position of lift, $|y_0 - y_1|$ is independent of the x-coordinate of the position of lift, so $|x_0 - x_1|$ and $|y_0 - y_1|$ are independent of each other. When there are many people, the total horizontal walking distance and vertical distances are independent of each other too.

Let a_{mn} be the number of people in the region located in column m and row n.

Then the total walking distance for all people is

$$\begin{aligned} &\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}(|x_i - x_0| + |y_i - y_0|) \\ = &\sum_{i=1}^{m}\sum_{j=1}^{n}(a_{ij}|x_i - x_0|) + \sum_{i=1}^{m}\sum_{j=1}^{n}(a_{ij}|y_i - y_0|). \end{aligned}$$

$\sum_{i=1}^{m}\sum_{i=1}^{n}(a_{ij}|x_i - x_0|)$ and $\sum_{i=1}^{m}\sum_{i=1}^{n}(a_{ij}|y_i - y_0|)$ are independent of each other so we can find the minimum for each independently to find the minimum of the total.

Therefore, the position of the lift can be found this way: Consider one row of the mall as a region; find the total number of people in each row. By the previous discussion on the case "one dimensional shopping mall with one lift" we can find an ideal row number y_0 such that $\sum_{i=1}^{m}\sum_{j=1}^{n}(a_{ij}|y_i - y_0|)$ attains minimum. By the same way we can find the x-coordinate of the lift and hence the position of the lift.

2.3. Three dimensional shopping mall with one lift

Different levels in the shopping mall we are using for discussion have the same area. We add up the number of people in each level with the same row and column number to get a two dimensional shopping mall. By the discussion above we can find the position for the lift.

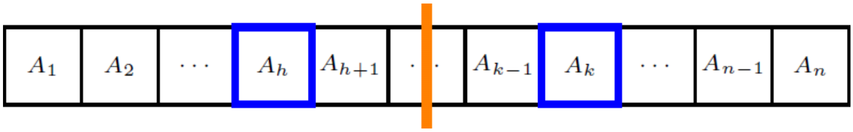
2.4. One dimensional shopping mall with two lifts

If a lift is at region 1 while another is at region n the people on the left of the mall will go to 1 while those on the right will go to n.

If we shift the lift rightward for 1 unit, then each people in regions 2 to region in the middle has to walk for 1 less unit but the people in region 1 has to walk 1 more unit. From the discussion of the case "One dimensional shopping mall with one lift" we know that the best position should equalize number of people on the left and on the right.

So when the lifts are at best positions regions h, k, number of people on the left of h equals the number of people who will go to lift h on the right, number of people who will go to k on the left of k equals the number of people on the right.

Existence of the ideal position for two lifts

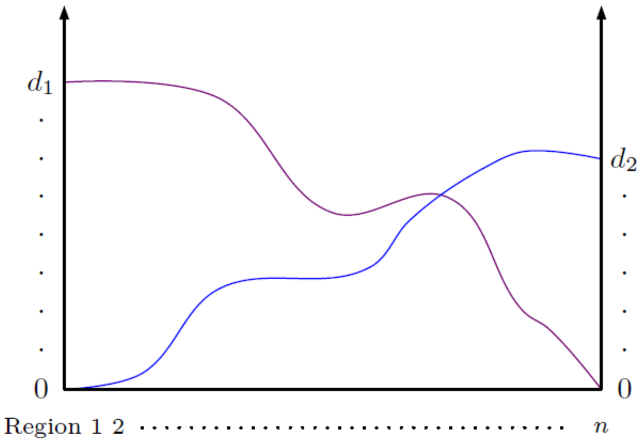


In the above figure, the orange line is the middle point between the lifts.

First, we choose a suitable position for the orange line. If it is possible to make the distance between the lift on the left and the orange line and that between the lift on the right and the orange line equal, then there exists the ideal positions for the two lifts.

Let distance between the lift on the left and the orange line be d_1 and the distance between the lift on the right and the orange line be d_2. Consider the extreme case with the orange line on the leftmost position of the mall. Then $d_1 = 0$, d_2 attains maximum.

Shift the orange line rightwards to the rightmost position. Then $d_2 = 0$, d_1 attains maximum.

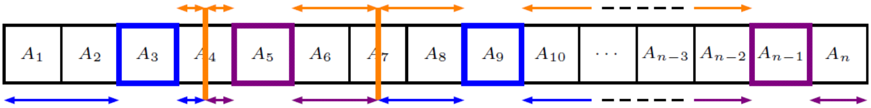


Orange line position

No matter how the distances d_1 and d_2 varies, as one increases from 0 onwards to a maximum and the other decreases from its maximum to 0, there must be a position at which $d_1 = d_2$.

The ideal position for m lifts

As a conclusion, for m lifts in a 1 dimension mall, the following is the ideal case.



In the figure, orange lines are the middle positions between two consecutive lifts. The orange lines divide the shopping mall into m regions. In each region, people will only use the lift in the middle of the region. Blue squares are the positions of the lifts. In each region, the position of the lift divides the number of people into two parts with one half on each side.

Computer Program

Base on the existence of the best arrangement of 2 lifts in a one dimensional mall, we have written a PASCAL program to find it.

```
program twolift;
var p,i,j,n,l,r,pr,pl,temp_sum,left_sum,right_sum:integer;
    a:array[1..1000] of integer;
    input,output:text;
begin
assign(input,'input3.txt');
assign(output,'output3.txt');
reset(input);
n:=0;
while not(eof(input)) do begin
    n:=n+1;
    read(input, a[n]);
    end;
close(input);
for i:=0 to n do begin
    left_sum:=0;
    right_sum:=0;
    for j:=1 to i do
        left_sum:=left_sum+a[j];
    for j:=i+1 to n do
        right_sum:=right_sum+a[j];
    temp_sum:=0;
    for j:=1 to i do begin
        if ((temp_sum<=left_sum/2) and (temp_sum+a[j]>left_sum/2))
            then l:=j;
        temp_sum:=temp_sum+a[j];
        end;
    temp_sum:=0;
    for j:=i+1 to n do begin
        if ((temp_sum<=right_sum/2) and (temp_sum+a[j]>right_sum/2))
            then r:=j;
        temp_sum:=temp_sum+a[j];
        end;
```

```
    if i-l+1=r-i then begin
            pl:=l;
            pr:=r;
            p:=i;
            end;
    end;
rewrite(output);
for i:=1 to n do
    if (i=pl) or (i=pr) then write(output,'# ')
        else write(output,'- ');
close(output);
end.
```

3. Conclusions

In this project, we aimed at finding the best positions for installing lifts. In particular, we have shown

1. In a one dimensional shopping mall, a lift should be installed at a position which divides number of people into two equal halves
2. In a two dimensional shopping mall, using the result from the previous case the best positions for 1 lift is one that divides number of people into two equal halves above and below, and at the same time on the left and on the right
3. In a three dimensional shopping mall, we add up the number of people in each level with the same row and column number to get a two dimensional shopping mall. By the discussion above we can find the position for the lift.
4. When there are two lifts to be installed in one dimensional shopping mall, we cannot find a simple algorithm to find the best positions but we can use computer program to do it.
5. The conditions that an arrangement of lifts has to satisfy if it is the best.

In fact, though we thought about our problem in a shopping mall, our findings are more applicable for locating emergency exits and staircases in a commercial building where the numbers of people in different regions are more or less constant. Moreover, the case one dimensional shopping mall with m lifts can be used for locating bus stops in a street along which population distribution is known.

4. Acknowledgement

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