## Acknowledgements

I first heard about Hang Lung Mathematics Awards (HLMA) in 2004 from a newspaper article about its launch. I was already a university student at that time, and my first reaction after reading the news was that I was too old to be qualified to participate in HLMA. Time flies and 17 years have passed. The year 2021 marks a new chapter for HLMA as Hang Lung Properties forms a new partnership with HKUST for HLMA. It is totally beyond my expectations that after all these years, I can eventually participate in HLMA – in a different role, of course.

My gratitude first goes to Mr. Ronnie C. Chan and Professor Shing-Tung Yau who have done so much to enrich mathematics education and the research atmosphere in Hong Kong by initiating HLMA almost two decades ago. I would also like to thank the former co-organizer CUHK, especially Professor Tom Wan and Professor Thomas Au, for their commitment and dedication to HLMA over the past 17 years, and for sharing with us their experience and sophistication in running this competition. Their contributions and support are indispensable for the success of this year's HLMA.

HLMA is reputed for its intellectual excellence. The adjudication process is very rigorous, and is governed by the Scientific Committee, chaired by Wolf Prize Laureate and world-renowned mathematician Professor Richard Schoen. Every submitted paper was first screened by the Screening Panel, and was then sent out to three professional reviewers who work in the field related to the submitted paper. The Scientific Committee then selected 14 strong teams to participate in the oral defense. I would like to thank every member of the Scientific Committee and the Screening Panel, and the 40 reviewers for upholding the integrity and high academic standard of HLMA.

2021 HLMA has also been benefitted from the excellent guidance of the Steering Committee, chaired by Nobel Prize Laureate Professor George F. Smoot, for widening the recognition of HLMA not only in the education sector, but the community as a whole. My thanks also go to the Executive Committee chaired by Professor Shing Yu Leung, the secretariat Ms. Niki Chan, and the team led by Ms. Susan Wong and Mr. Kevin Ding from Hang Lung Properties who are in-charge of the administration, promotion, and organization of HLMA. I also thank a group of hard-working HKUST undergraduate students: Tsz Kin Chan, Anson Chung, Howard Li, Jiaming Liu, Thanic Nur Samin, Burton Wong, Horrison Wong, and Thomas Yeung, who worked around the clock to typeset and reformat this collection of winning papers.

The publishing of this compilation of winning papers of the 2021 HLMA has been made possible by the donations from Hang Lung Properties. It is the ninth volume of winning papers being published, and I hope that this volume will continue to be welcomed by schools and the community.

"The quality of the papers in the competition was very impressive to me. The breadth of topics and the depth and clarity of solutions given were very high, certainly at the level of top high-school and beginning years of college students in the U.S. The ability to rigorously model and isolate the essential features of a complex problem and to develop the language and methods to solve it efficiently are tools which will serve you well in the future."

## Professor Richard Schoen

Chair, 2021 Hang Lung Mathematics Awards Scientific Committee 2017 Wolf Prize Laureate in Mathematics Stanford University University of California, Irvine

"The HLMA is a unique high school research competition that challenges students to approach problems in a scientific way and find solutions to the research process.

The stamina and skills required are quite different from some classroom learning and test-based competitions, and the experience can certainly give students an edge in their future studies."

## Professor George F. Smoot

Chair, 2021 Hang Lung Mathematics Awards Steering Committee 2006 Nobel Laureate in Physics The Hong Kong University of Science and Technology